Antonine Physics AS

Topic 5 Mechanics

How to Use this Book

How to use these pages:

- This book intended to complement the work you do with a teacher, not to replace the teacher.
- Read the book along with your notes.
- If you get stuck, ask your teacher for help.
- The best way to succeed in Physics is to practise the questions.

There are many other resources available to help you to progress:

- Web-based resources, many of which are free.
- Your friends on your course.
- Your teacher.
- Books in the library.

This is an electronic book which you can download. You can carry it in a portable drive and access it from your school's computers (if allowed) as well as your own at home.

This is a long topic. You may need to refer to it even in your second year (A-Level).

Mechanics is about the study of forces, energy, and movement. It is the same discipline as what is taught in the first year Mathematics syllabus, but in these notes, I apply it more in the context of Physics. Many concepts in engineering are based on what we will learn here.

We will look at how **forces** act, especially in **equilibrium** - important in structures.

We then go on to look at **moments** and consider their significance in bridges and stability.

We will look at **motion in a straight line** and consider **Newton's Laws of Motion**. These were written 350 years ago but are just as valid today as they were then.

We then go on to **motion in a gravity field**, including free fall, terminal velocity, friction, and drag.

We will look at the concepts of momentum, and how it can be used to explain what happens when **balls are struck**, **collisions**, and **explosions**.

We then go on to consider how **work** is done when a force is moved, then move on to **efficiency** and **conservation of energy**.

TOPIC 5 MECHANICS

Topic 5 Mechanics

Part 1 Forces

Tutorial 5.01 Vectors and Scalars

All Syllabi

Contents

5.011 Scalars and Vectors

Central to the study of mechanics is the idea of the vector quantity that not only has a value or magnitude, but direction as well. Examples include:

- Acceleration.
- Force.
- Velocity.

Any quantity that does not specify a direction is a **scalar**, examples of which include:

- Energy.
- Temperature.

Vector	***Scalar***	***Unit***
Displacement	Distance	Metres (m)
Velocity	Speed	Metres per second (m s^{-1})
Acceleration		Metres per second^2 (m s^{-2})
Momentum		Newton seconds (N s)
Force		Newtons (N)
	Work, Energy	Joules (J)
	Voltage	Volts (V)
	Temperature	Degrees Celsius (°C)
	Frequency	Hertz (Hz)

Notice that the **units** are the same, regardless of whether they are vectors or scalars.

Note:

- Some vectors can be used as scalar quantities.
- There are some scalar quantities, e.g. temperature, that have no vector equivalent.
- The shorthand for metres per second is either written ms^{-1} or m/s. Either is acceptable, although $m\ s^{-1}$ is much better.
- Some vectors have the same units as scalars. For example, velocity has units of $m\ s^{-1}$ and speed has units $m\ s^{-1}$.
- Work is the product of two vectors (Work = Force × distance moved in direction of force) but it is a **scalar**.
- You will sometimes be asked to find the **magnitude** of the vector. This means that you just put down the **value**.
- In some situations, direction is absolutely critical.

Remember:

- The product of a vector × scalar is a vector:
- Momentum = mass × velocity.
- Momentum is vector, mass is scalar, velocity is a vector.

- The product of two vectors is a scalar.
- Potential energy = mass × gravity × displacement
- Energy is a scalar, mass is a scalar, while gravity and displacement are vectors.

- The square of a vector is a scalar.
- Displacement squared (= area) is always positive.

5.012 Adding Vectors

Suppose we apply two forces to an object, represented by the yellow blob in *Figures 1* and *2*. We can represent any body, however complicated it is in real life, by a simple shape like a circle. We refer to these as **free body diagrams**.

If the force vectors of 3N and 4N are in the same direction, they simply **add** together (*Figure 1*).

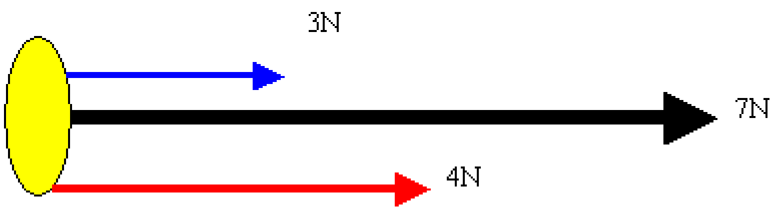


Figure 1 Vectors in the same direction simply add up

The heavy arrow indicates the **resultant** force.

If the vectors are in opposite directions, we **subtract** (*Figure 2*).

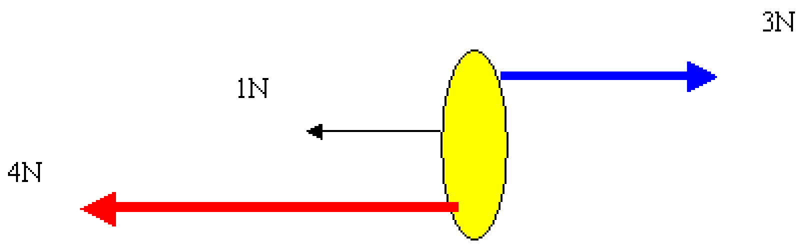


Figure 2 Vectors in opposite directions subtract

We can see that the resultant is now just 1 N.

If the two vectors are at 90° use **Pythagoras' Theorem** (*Figure 3*).

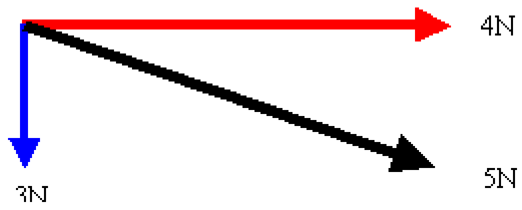


Figure 3 Using Pythagoras to add vectors at 90°

$\text{Resultant}^2 = (3\text{ N})^2 + (4\text{ N})^2 = 9\text{ N}^2 + 16\text{ N}^2 = 25\text{ N}^2$.

Resultant = $\sqrt{(25\text{ N}^2)}$ = **5 N**

To work out the angle we use the **tan** function:

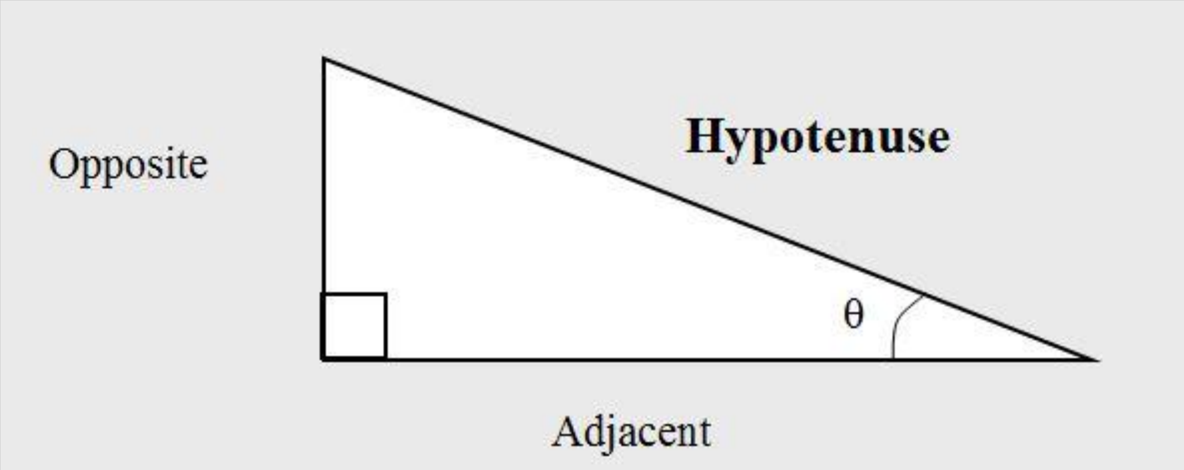
$$\tan\theta = ¾ = 0.75$$

$$\theta = \tan^{-1}(0.75) = \mathbf{36.9^\circ}$$

Maths Window

You need to understand **trigonometrical** functions in order to resolve vectors, and to work out the angles that the resultants make. You will have done this in Maths at school but in case you weren't listening at the time...

Sines, cosines, and tangents are ratios between the sides of a triangle for a given angle.



The symbol θ is "theta", a Greek letter 'th', which is used to denote angles.

- $\sin\theta = \dfrac{\text{opposite}}{\text{hypotenuse}}$

- $\cos\theta = \dfrac{\text{adjacent}}{\text{hypotenuse}}$

- $\tan\theta = \dfrac{\text{opposite}}{\text{adjacent}}$

- $\tan\theta = \dfrac{\sin\theta}{\cos\theta}$

There are all sorts of different mnemonics to help you understand these relationships. A rule of thumb is that if you get a sine or cosine that is greater than 1, you've done it wrong.

5.013 Vectors not at Right Angles

We can also add vectors that are not at right angles (*Figure 4*).

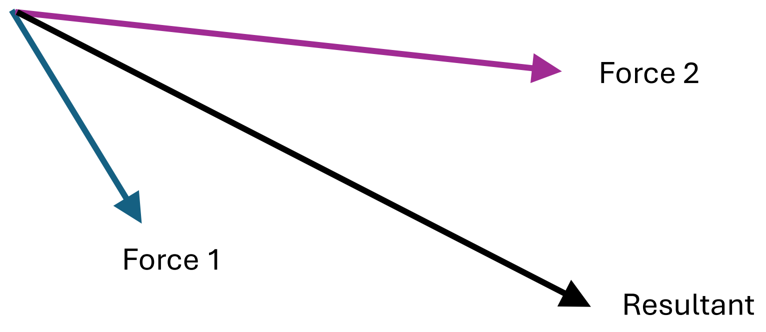


Figure 4 Vectors that are NOT at right angles

In the picture above we can see the resultant of two forces that are not at right angles. We can show that they make a vector triangle by moving Force 1 (*Figure 5*).

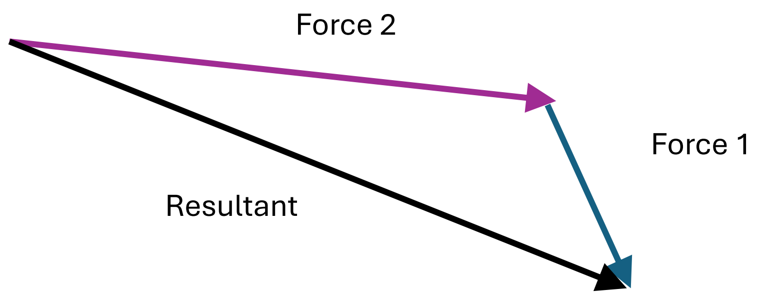


Figure 5 Three vectors making a triangle

Alternatively, we can use a parallelogram of forces as shown below in *Figure 6*:

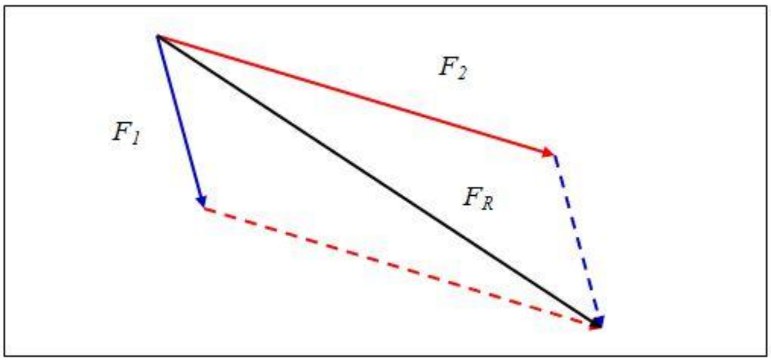
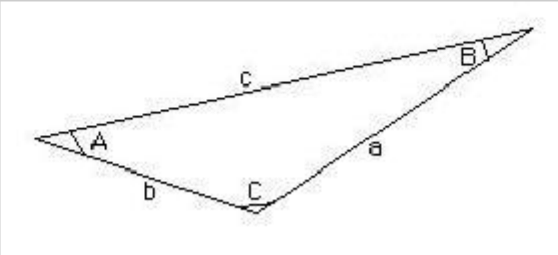


Figure 6 A parallelogram of forces

The resultant can be worked out by accurate drawing. Or you can use the **cosine rule**.

Maths Window

The cosine rule can be used to work out the resultant of two vectors that are not at right angles.



$$a^2 = b^2 + c^2 - 2bc\cos A$$

$$F_2^2 = F_1^2 + F_R^2 - 2F_1F_R\cos\theta$$

At A-level, you would only have to add vectors at 90 degrees to each other. If there were vectors as shown above, then you would normally be expected to use **accurate drawing**. The question would tell you to do accurate drawing, although I am sure that if you got the right answer from the cosine rule, you would be awarded full credit.

It is easier to add vectors that are not at right angles using **resolution of vectors**.

5.014 Resolution of Vectors

We can resolve any vector into two **components** at **90°** to each other. They are called the **vertical** and the **horizontal** components. It is important to remember that the two vectors are **independent** of each other, meaning that the value of the horizontal component has no bearing on the value of the vertical component. See *Figure 7*.

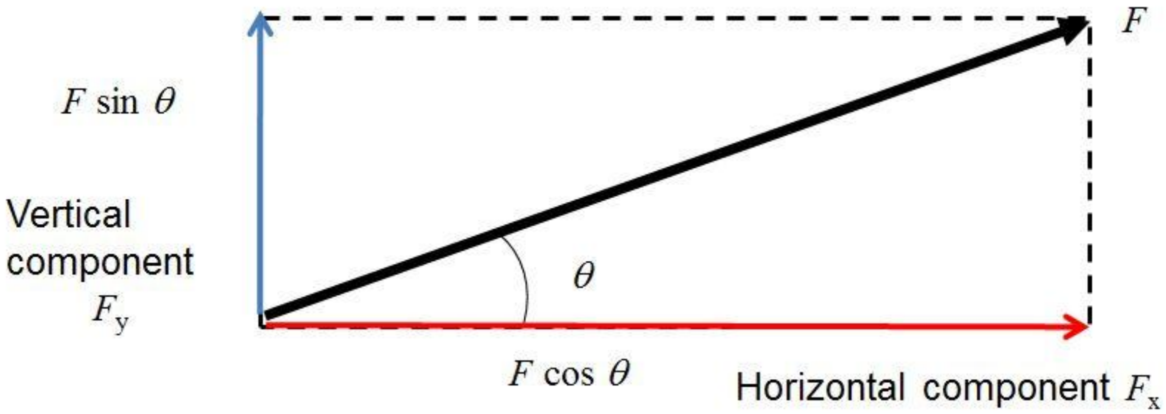


Figure 7 using vertical and horizontal components to resolve a vector.

$$F_x = F \cos \theta$$ .... *Equation 1*

$$F_y = F \sin \theta$$ .... *Equation 2*

Any vector in any direction can be resolved into horizontal and vertical components. These can be calculated by **accurate drawing** or **trigonometry**.

Consider a car going up a hill (*Figure 8*).

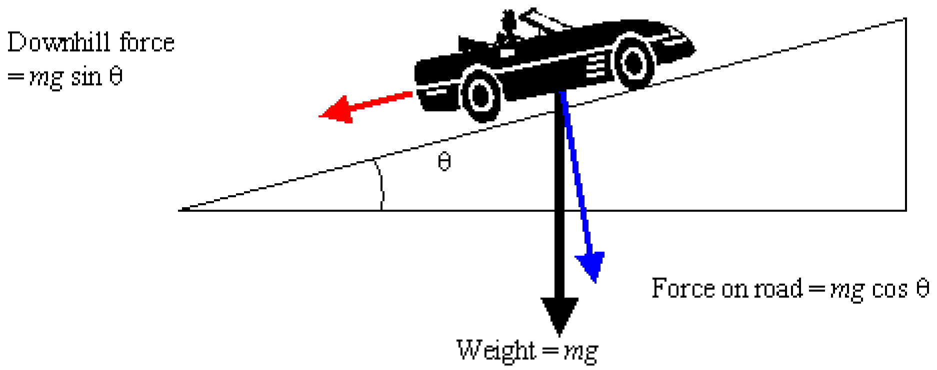


Figure 8 Forces acting on a car going uphill

The angle of the hill is θ degrees. We must note that the weight (given by the mass in kilograms × acceleration due to gravity) is always pointing **vertically down**. Acceleration due to gravity can be taken as 9.8 m s^{-2} and the force of gravity is 9.8 N kg^{-1}. We can resolve the vectors, remembering that the weight acting vertically **is the resultant force**.

The force vectors are arranged like this (*Figure 9*).

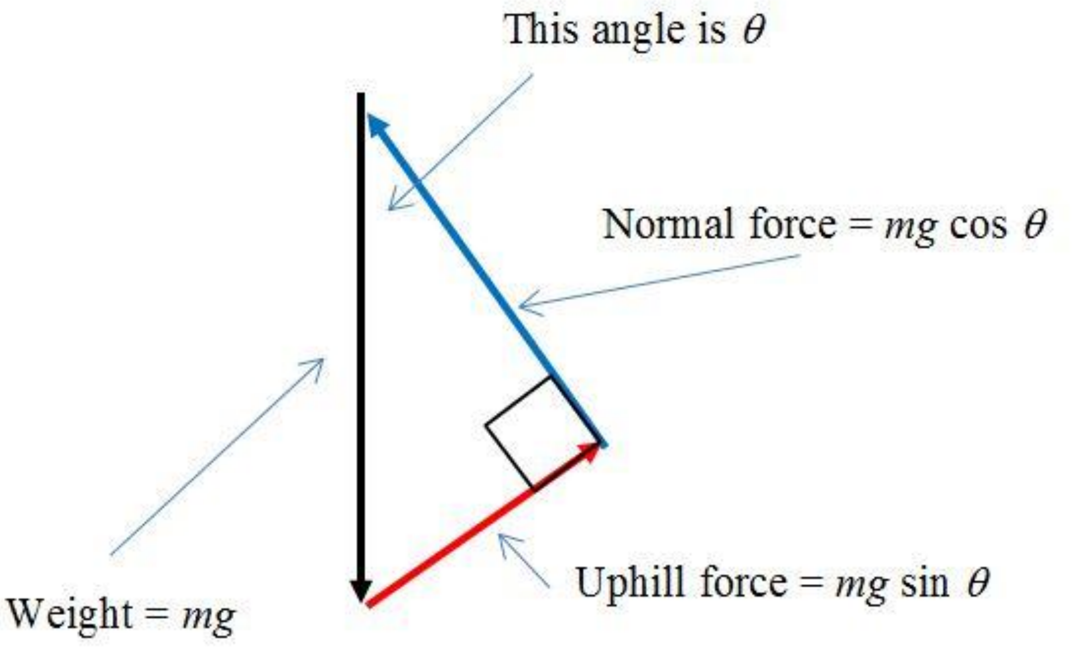


Figure 9 Force vectors acting on an object on a slope.

Note that:

- The weight vector forms the **hypotenuse** of the triangle.
- Simple geometry shows that the angle between the weight and the normal is $\theta°$.
- **Normal** means at 90 degrees to. The normal force is the force on the road.

Remember:

- **Mass** is how much material there is in an object. It is measured in kilograms (kg).
- **Weight** is a force measured in newtons (N). It is the product of mass and the acceleration due to gravity. It is the force exerted by a mass due to gravity.

It is depressing how many students write weight in kilograms. Watch out for this bear trap!

You cannot talk of vertical and horizontal components of a vertical vector. The components are perpendicular to each other.

5.015 Adding vectors by resolution

When vectors are not at right angles, it is much easier to resolve each vector into vertical and horizontal components (*Figure 10*).

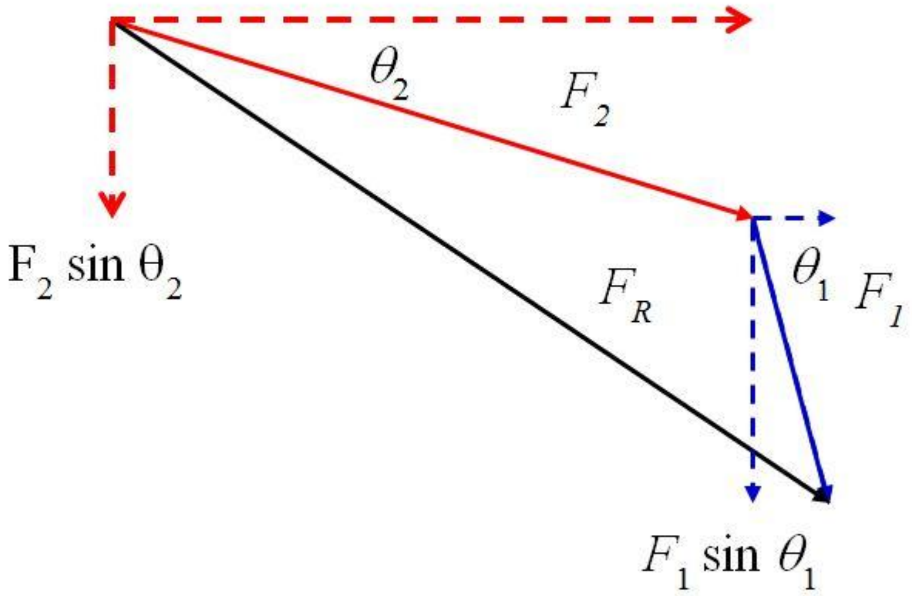


Figure 10 Resolving vectors that are not at 90°.

The components add up like this (*Figure 11*):

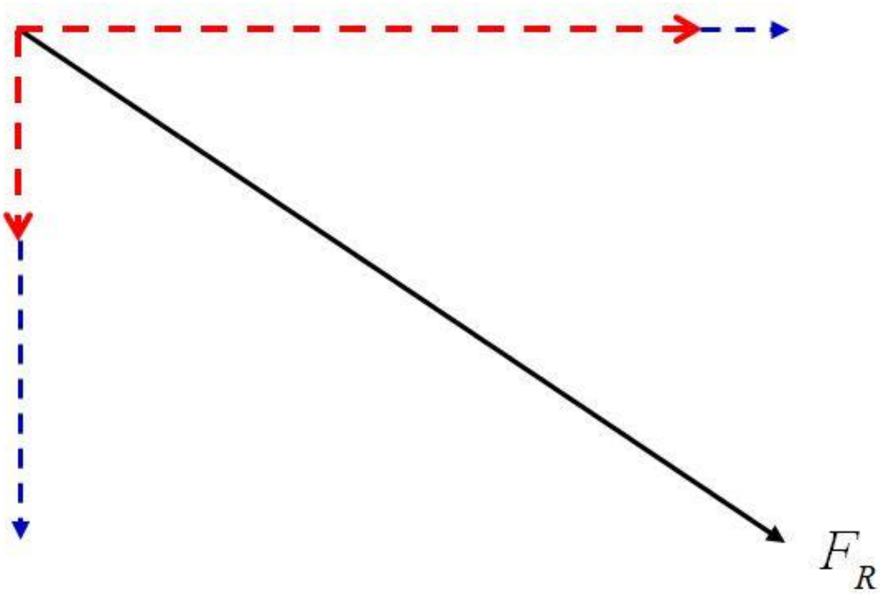


Figure 11 Adding the vertical and horizontal components.

Add the horizontal vectors together:

$$F_H = F_1 \cos\theta_1 + F_2 \cos\theta_2$$

.... *Equation 3*

Add the vertical vectors together:

$$F_V = F_1 \sin\theta_1 + F_2 \sin\theta_2$$

.... *Equation 4*

Finally, we do the vector addition:

$$F_R^2 = {F_V}^2 + {F_H}^2$$

.... *Equation 5*

5.016 Resolving Vectors in 3-dimensions (Extension only)

It is useful to be aware of this, but it won't be in the exam.

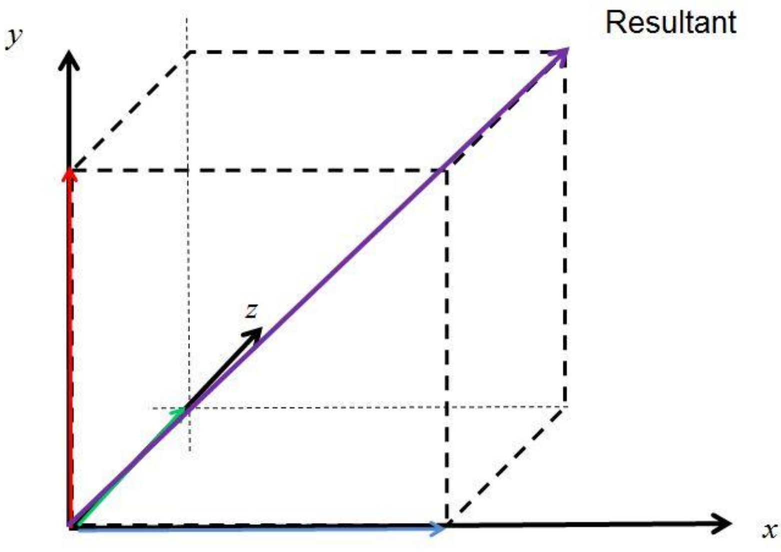


Figure 12 Resolving vectors in three dimensions.

The resultant vector is the vector sum of the x, y, and z components:

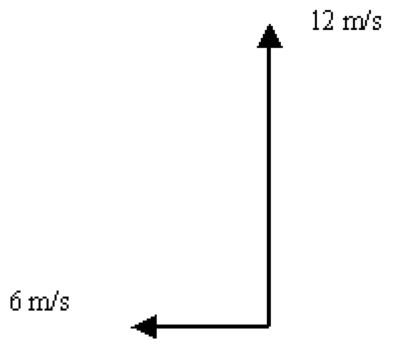
$$R^2 = x^2 + y^2 + z^2$$

.... *Equation 6*

Tutorial 5.01 Questions

5.01.1

What are the resultants of these vectors?



5.01.2

What are the angles that the resultants make to the vertical in the previous question?

5.01.3

The car in *Figure 8* has a mass of 1100 kg, and the angle of the slope is 10°.

Calculate:

(a) the weight of the car,

(b) the force on the road,

(c) the downhill force.

(Use g = 9.8 N kg^{-1})

Tutorial 5.02 Equilibrium for Coplanar Forces	
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5.021 Free body diagrams

Real objects are complicated things. In mechanics, we model them as **point masses**, which makes things rather simpler. We do this as a **free body diagram** which shows all the forces acting on an object as acting on the **centre of mass**.

Consider this diagram here (*Figure 13*):

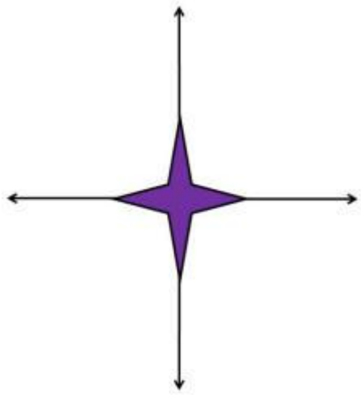


Figure 13 Forces acting on a random object.

Becomes (*Figure 14*):

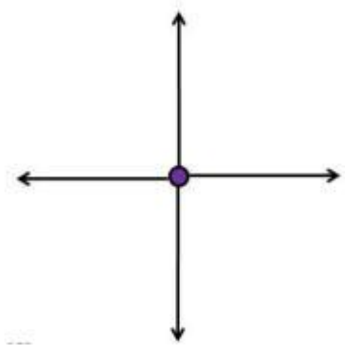


Figure 14 A free body diagram

We treat all objects as if they are **point masses**, so that we don't have to do a very complicated diagram. All forces act on a single point.

5.022 Balanced Forces

Forces in **equilibrium** mean that they are **balanced**. **Coplanar** forces act in the same plane. Two balanced forces are equal in **magnitude** but opposite in **direction** to the other.

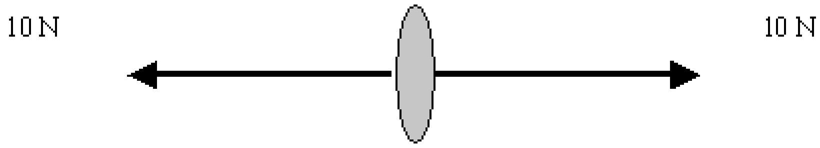


Figure 15 balanced forces acting on an object.

We can see easily from the **free body diagram** that the resultant force is **zero**.

This does NOT mean that there are no forces. It means that the directions and values of the forces add up to zero.

This is true for any number of forces.

If an object is at rest, it means that it is subject to forces that **add up to zero**.

If the resultant force is zero, the system is in **equilibrium**.

If there is an overall resultant force, **movement**, or a **turning effect** will result. Strictly speaking, **acceleration** will result.

5.023 Triangle of Forces

If we are considering **three** coplanar forces in equilibrium, use the **triangle of forces rule**:

If 3 forces acting at a point can be represented in size or direction by the sides of a closed triangle, then the forces are in equilibrium, provided their directions can form a closed triangle.

Coplanar means that the forces are in the same **plane** (flat surface) and **equilibrium** means that the forces are **balanced**. Remember that the total **resultant** force is **zero**. That does not mean that there no forces; the forces' values and directions mean that the total force is zero.

This means that the forces can follow each other round a triangle (*Figure 16*).

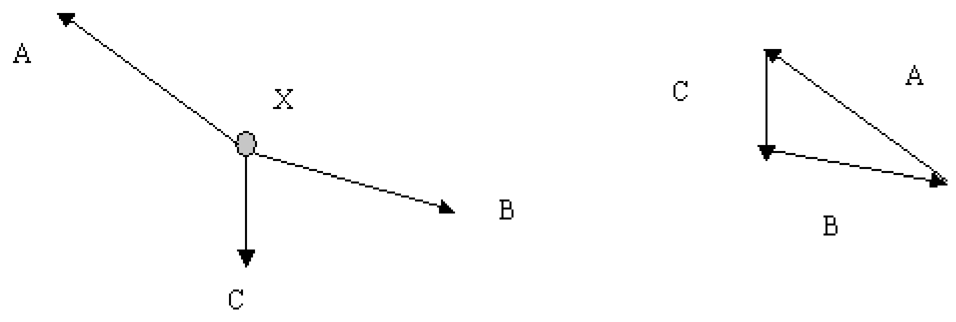


Figure 16 Three forces in equilibrium form a closed triangle.

Notice how:

- The forces form into a closed triangle.
- The directions of the forces go round the triangle.

Consider this (*Figure 17*):

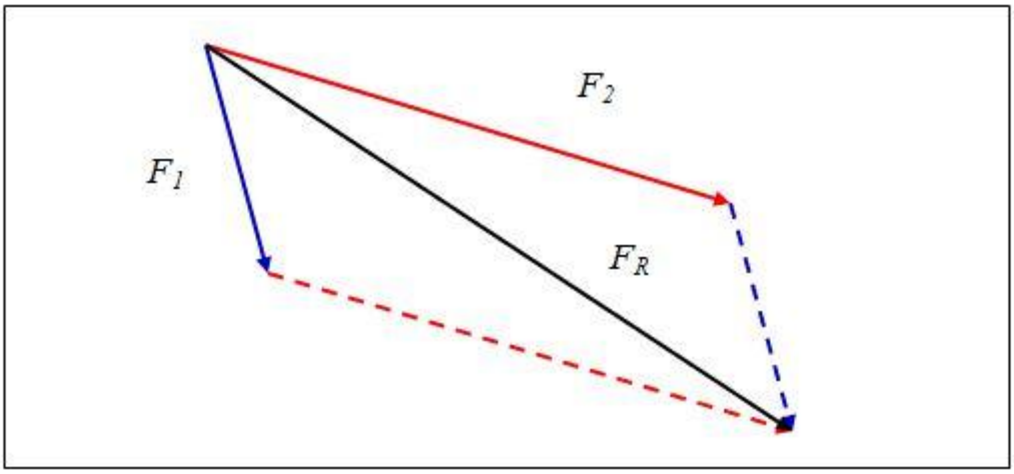


Figure 17 These forces are not in equilibrium.

The forces F_1 and F_2 make a **closed triangle**, but they are NOT going nose-to-tail. These forces are NOT in equilibrium. F_1 and F_2 make up a **resultant** F_R (*Figure 18*).

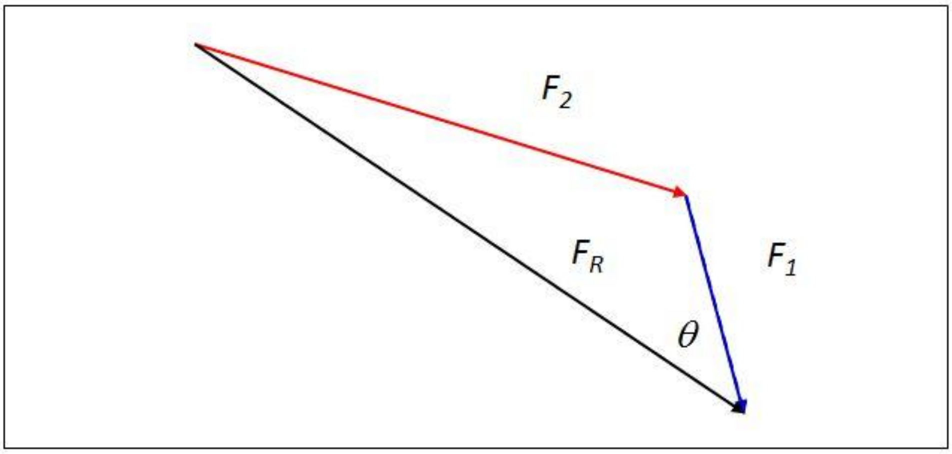


Figure 18 These two forces make a resultant force.

Remember that you must consider the **directions** of the forces as well as the values (**magnitudes**) and **angles**.

This is a very common bear-trap.

Now let's reverse the directions of F_1 and F_2 but keep the **magnitudes** and the angles the same (*Figure 19*).

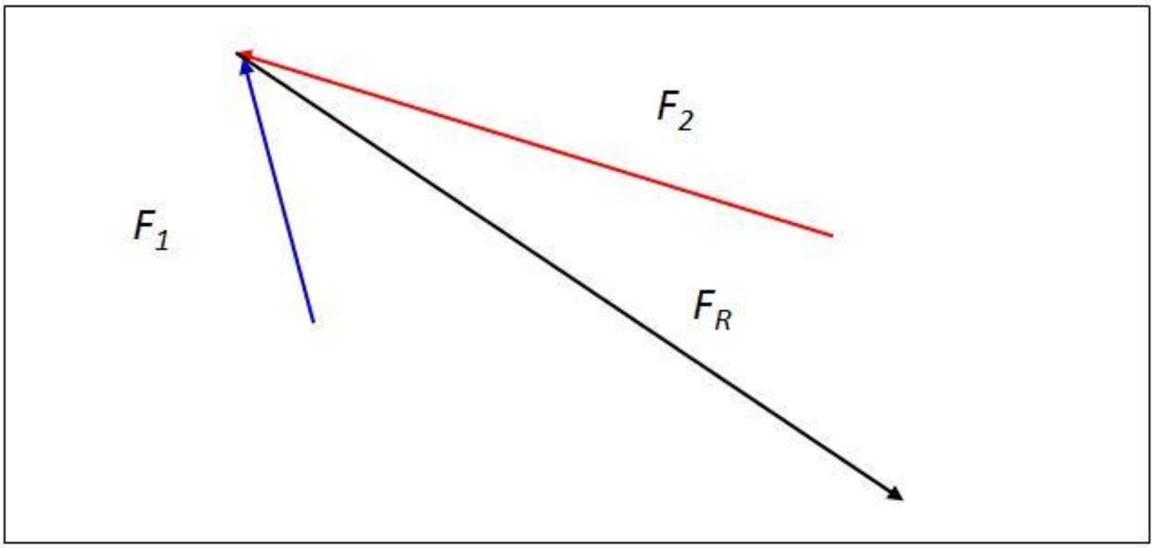


Figure 19 Reversing the directions of F_1 and F_2

If we move F_1 we find that the three forces form a **closed triangle**, and the arrows go **nose-to-tail** (*Figure 20*).

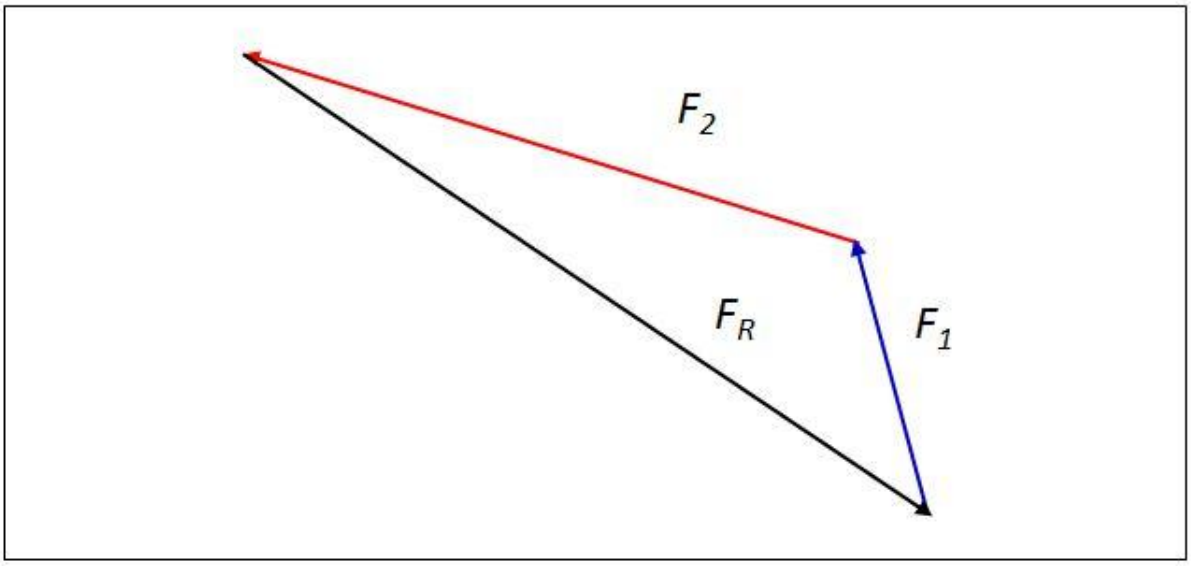


Figure 20 Three forces in equilibrium form a closed triangle.

This is what the three forces look like when acting on a single point (*Figure 21*).

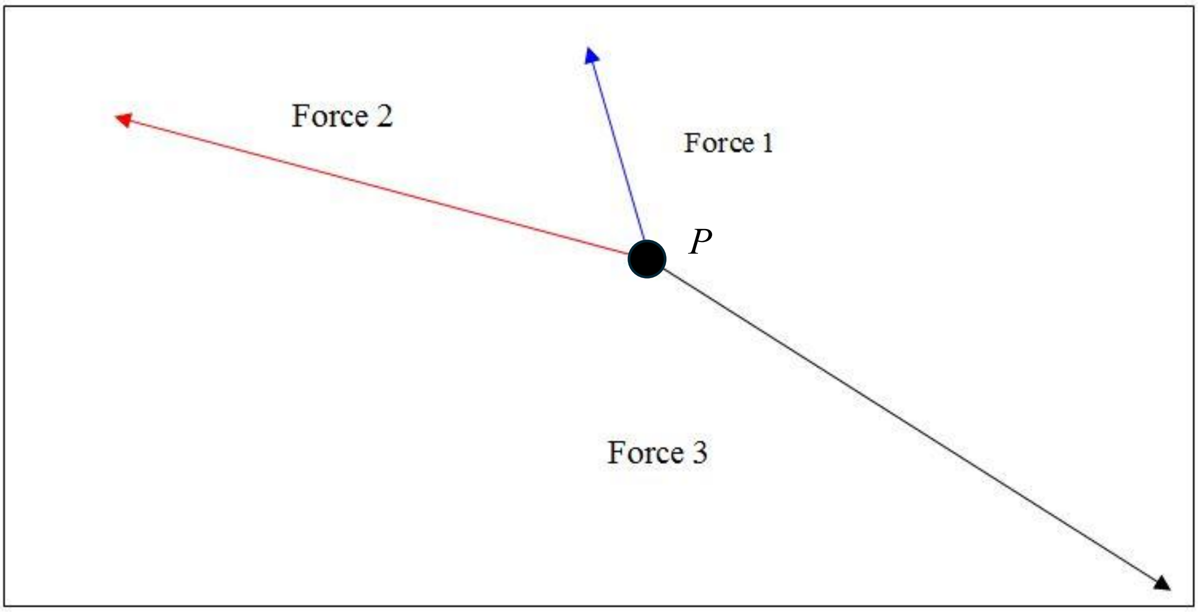


Figure 21 Three forces in equilibrium acting on a single point P.

This means that the system is **in equilibrium**.

The **resultant** of Forces 1 and 2 will be of the same **magnitude** as Force 3, but in the **opposite** direction. This means that the forces will be balanced, and the resulting force is **zero**. If any force vector **polygon** forms a **closed loop**, the forces are in equilibrium. This is the polygon of forces rule and it's true for any number of forces.

The study of forces in equilibrium is called **statics**, and is an important consideration when structures are designed. In **statics**, remember that all forces add up to zero. That does not mean that there are no forces. The forces balance each other out. These situations are for external forces acting on an object, but within an object, all forces sum to zero. Otherwise, the object would fly apart.

In statics problems, we need to know how to resolve forces, which can be done by:

- accurate drawing.
- use of trigonometry.

5.024 Resolving the Vectors by Accurate Drawing

Consider three forces acting in equilibrium (*Figure 22*):

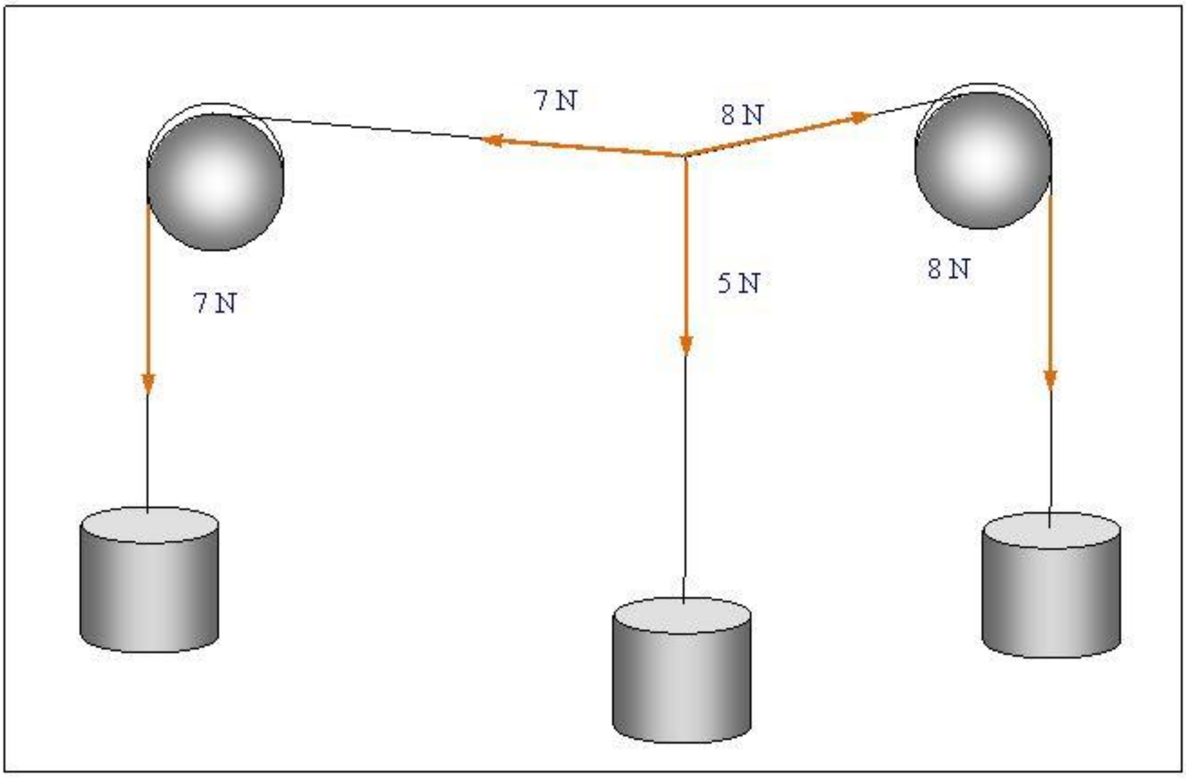


Figure 22 Three forces in equilibrium.

We see that the forces (from the weights) form a closed triangle, and the directions form a closed loop. Therefore, the forces are balanced. We can show this as a triangle of forces (*Figure 23*).

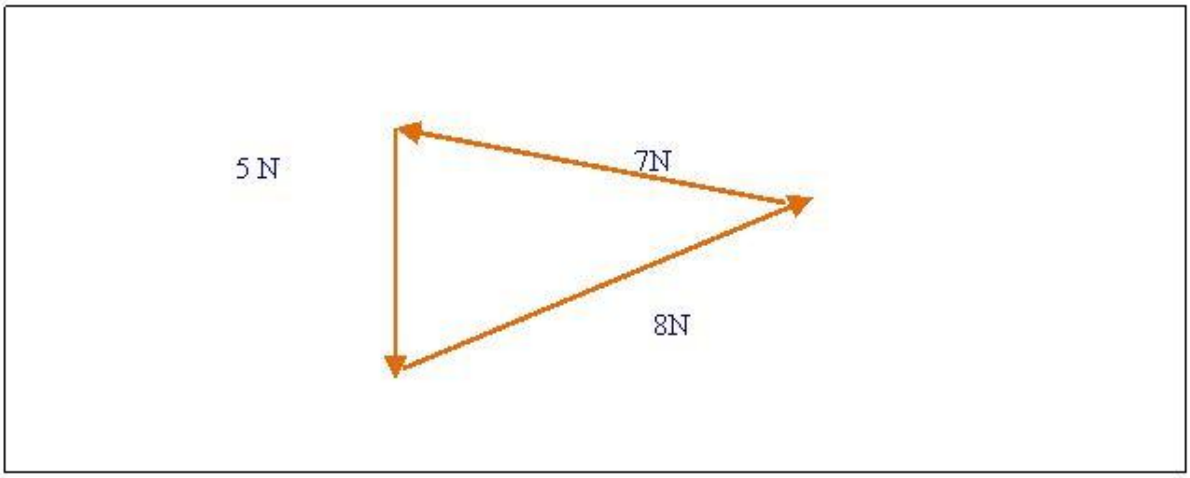


Figure 23 A triangle of forces.

Here some rules for accurate drawing:

1. Choose a scale (e.g. 2 cm = 1 N)
2. Use graph paper.
3. Use a sharp pencil.
4. Use a compass.
5. Use a protractor if angles are mentioned.
6. Draw the arrows in the direction specified.

Here is the equilibrium situation above represented by **accurate drawing** (*Figure 24*).

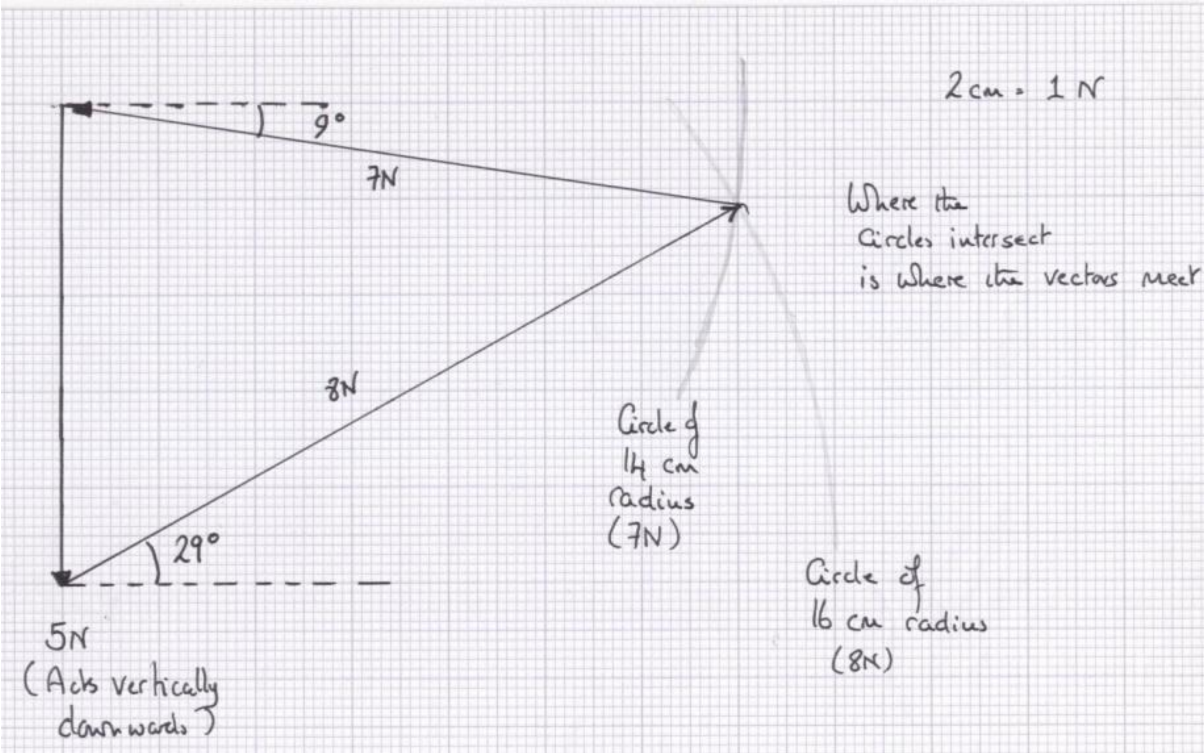


Figure 24 Solving a triangle of forces problem using accurate drawing.

The angles are measured with a protractor to give the values shown.

Accurate drawing is not that accurate.

You are doing well if you can get an answer to 2 significant figures.

There is a range of acceptable answers in an examination marking scheme. Do not blindly regurgitate anything your calculator produces.

***In the exam*:**

Trigonometry gives better results, but if you have difficulty, use accurate drawing. You will gain full credit. Don't throw away marks by leaving a blank space.

Forces and vectors are a topic in which it is quite likely that you will have to apply physics principles and concepts to unfamiliar situations. So when you think "we didn't do this in class," it is one of the situations in which this skill is being tested.

You may also be asked to apply these ideas to technological situations, for example, an aeroplane towing a glider.

Often questions may be asked that cover several different topics, and it's an important skill to be able to bring principles from several different areas to bear on a problem.

5.025 Resolving Vectors by Trigonometry

The problem with accurate drawing is that of accuracy. If you get the answer to the nearest degree, you're doing well. And accurate drawing is not easy. If you are challenged by measuring, resolution of vectors using **trigonometry** is the answer.

Any vector in any direction can be resolved into **vertical** and **horizontal** components at 90 degrees to each other (*Figure 25*).

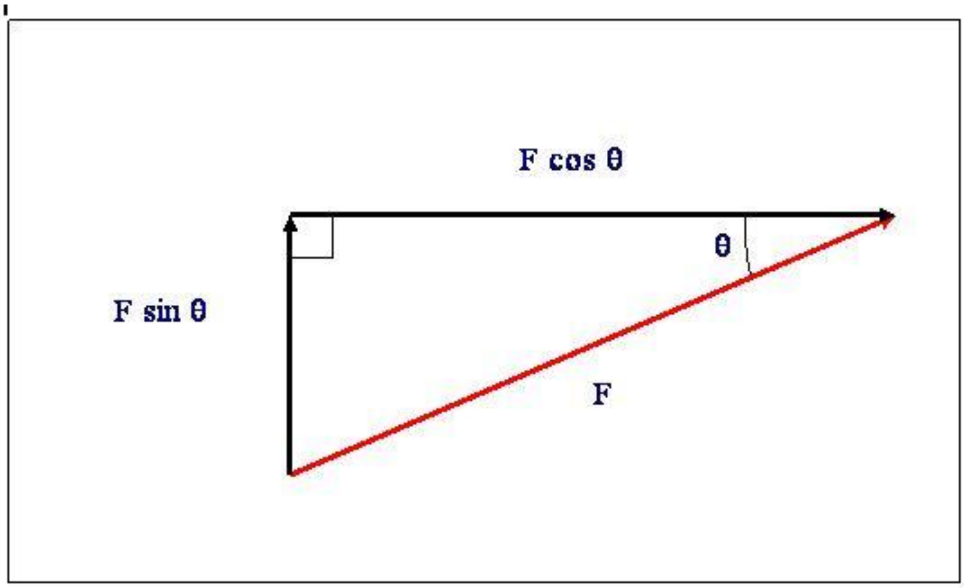


Figure 25 Any force vector can be resolved into vertical and horizontal components

For three forces in equilibrium, we can draw a **force vector diagram** (*Figure 26*).

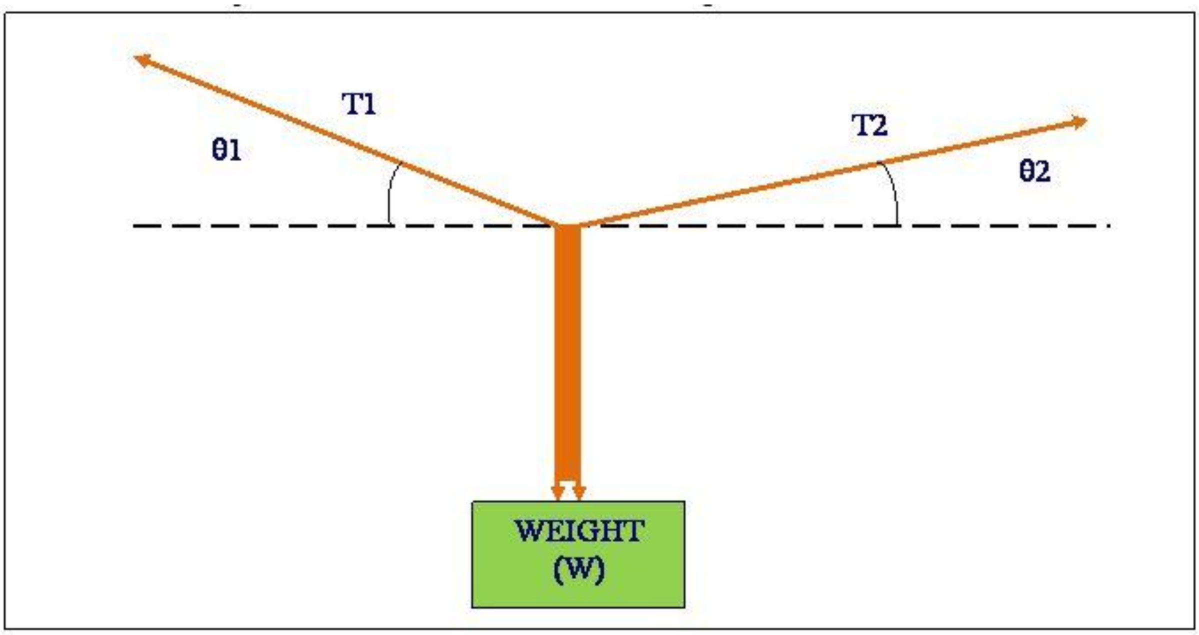


Figure 26 Force vector diagram.

For this situation we know that weight always acts **vertically downwards**. We can resolve the two other vectors into their horizontal and vertical components:

1. T_1 resolves into $T_1 \cos \theta_1$ (horizontal) and $T_1 \sin \theta_1$ (vertical).
2. T_2 resolves into $T_2 \cos \theta_2$ (horizontal) and $T_2 \sin \theta_2$ (vertical).

We know that the three forces add up to zero, so we can say:

- $T_1 \cos \theta_1 + - T_2 \cos \theta_2 = 0.$
- This means that the forces are equal and opposite.
- $\Rightarrow \boldsymbol{T_1 \cos \theta_1 = T_2 \cos \theta_2}$
- $T_1 \sin \theta_1 + T_2 \sin \theta_2 + - W = 0$
- Weight is acting **downwards** and downwards is, by convention, **negative**.
- $\Rightarrow T_1 \sin \theta_1 + T_2 \sin \theta_2 = W$

Be careful that you don't assume that W is split evenly between $T_1 \sin \theta_1$ and $T_2 \sin \theta_2$. This is only true when the weight is half way between the ends.

Many students write $T_1 \sin \theta = mg$ which is wrong.

5.026 Forces acting symmetrically

Consider a mass m that is suspended, so that it hangs freely, half-way between two points X and Y. Its weight will be mg and it will cause a tension, T_1 in the left-hand part of the string and a tension T_2 in the right-hand part of the string, as shown (*Figure 27*).

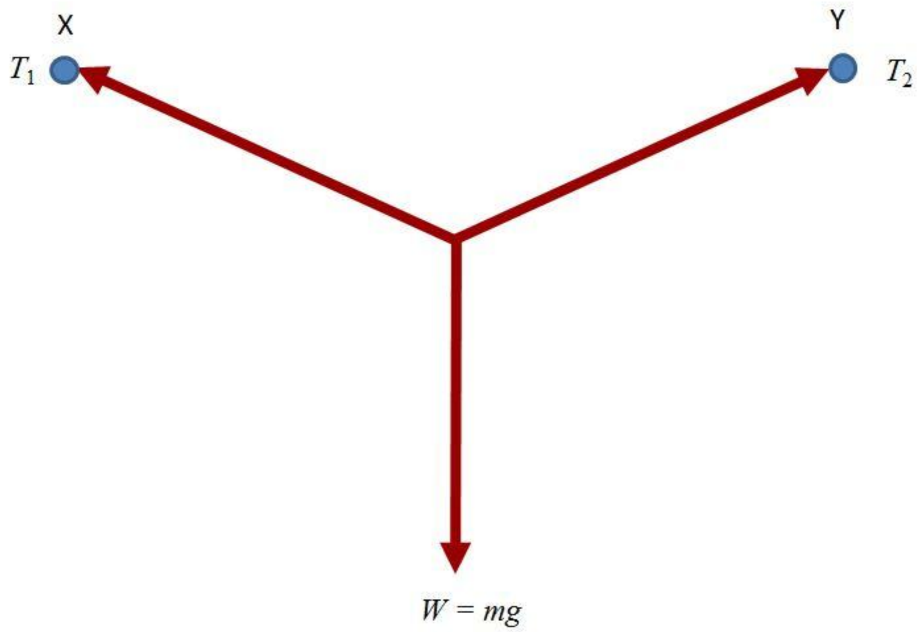


Figure 27 A symmetrical force diagram

We can show that this obeys the triangle of forces rule (*Figure 28*).

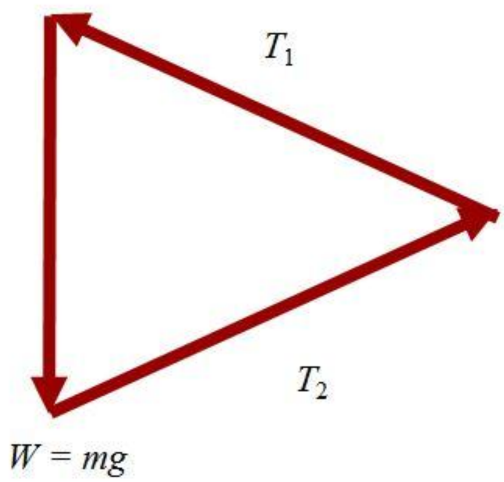


Figure 28 Triangle of forces

An isosceles triangle is formed. We can resolve the forces T_1 and T_2. This is a **symmetrical** situation. $T_1 = T_2$ (*Figure 29*).

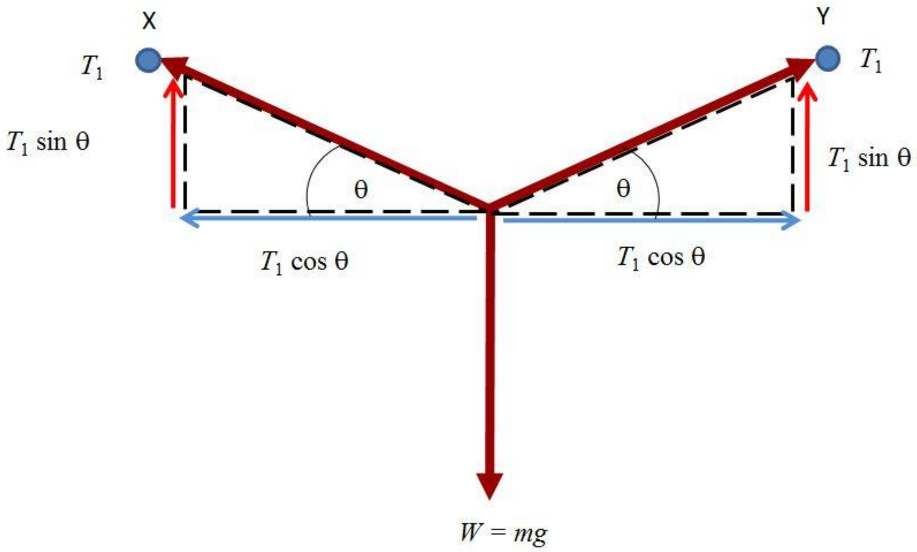


Figure 29 Resolution of forces in a symmetrical equilibrium

The horizontal components add up to **zero** because they are equal in magnitude, and in opposite directions.

The vertical components add up to *mg*, the vertical downwards force:

$$2T_1 \sin \theta = mg$$

.... *Equation 7*

Therefore:

$$T_1 \sin \theta = \frac{mg}{2}$$

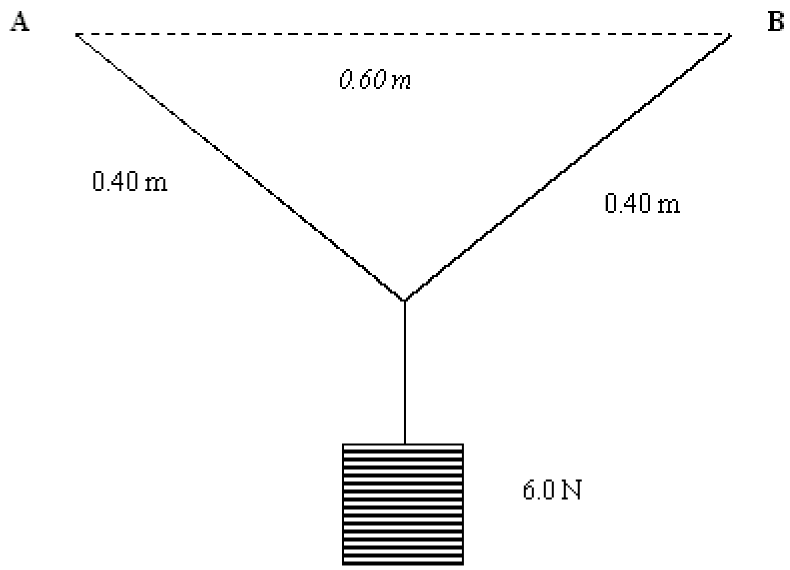
.... *Equation 8*

Many students write $T_1 \sin \theta = mg$ which is wrong.

Do NOT assume that any diagram drawn in an exam question is drawn to scale. The chances are that they are not.

Worked Example

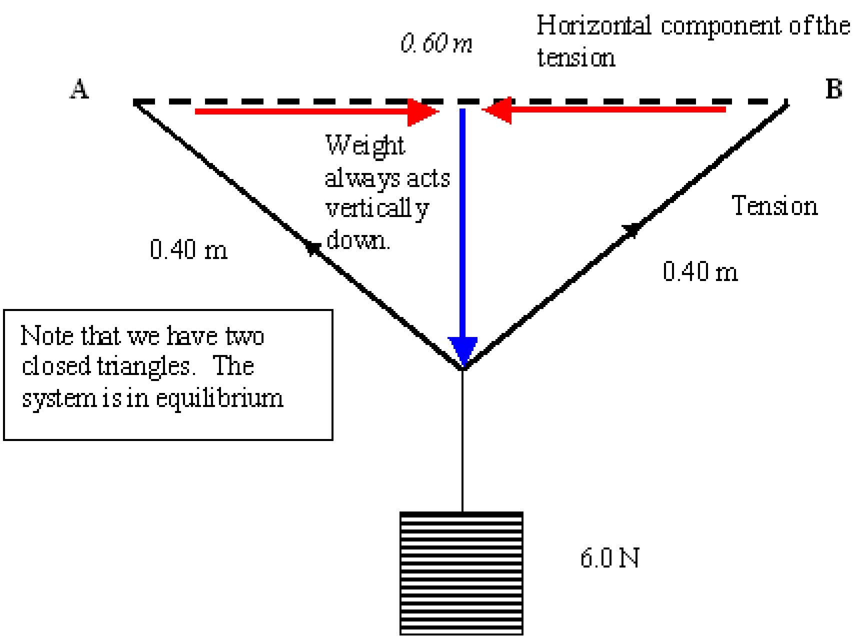
A 6.0 N weight is attached to a light string which is then tied to the midpoint of a second string of length 0.8 m. This string is suspended from two fixed points which are on the same horizontal line 0.60 m apart. The arrangement is shown below:

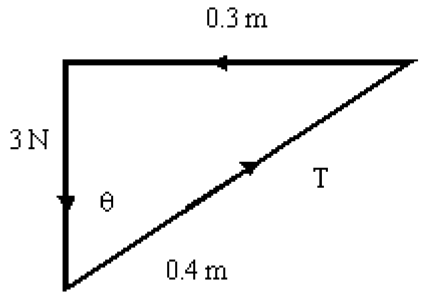


What is the angle between the two halves of the string?

What is the tension in each half of the string?

First of all, draw the forces and the directions:



The weight vector splits the set up into two right-angled triangles. The horizontal components are equal and opposite, so cancel each other out. The weight vector can be considered as the resultant of two downward vectors which add up the total downward force. Since this is a symmetrical system, each vector is 3 N. Each closed triangle looks like this: 
Now we can find *q*, the angle with the vertical. It is the **opposite**, and we know the **hypotenuse**, so we use the **sine function**. $$0.4 \sin \theta = 0.3$$ $$\sin \theta = 0.3/0.4 = 0.75$$ $$\theta = \sin^{-1} 0.75 = 48.6^{\circ}$$ We now can say that the angle between the two strings is 2 x 48.6° = <u>**97.2°**</u>
We can now work out the tension: The angle θ is the adjacent, so we use the cosine function. $$T \cos 48.6 = 3\text{N}$$ $$T = 3\text{N} \div \cos 48.6 = 3\text{ N} \div 0.661 =$$ <u>**4.54 N**</u>

At AS level, you are most likely to encounter symmetrical systems like this. If the system is NOT symmetrical, don't panic. Remember:

- The weight (downwards force) can be split into two force vectors.
- These add up to the weight.
- The horizontal force vectors add up to zero.
- If you are completely stuck, use accurate drawing!
- **Whatever you do, don't leave a blank!**

5.027 Non-symmetrical Equilibrium (Extension)

This is more challenging but is not that difficult. The key things to remember are that:

- The tension T_1 and T_2 will be different.
- The angles θ_1 and θ_2 will be different.

Consider a mass, m, which is being suspended freely between two points X and Y, but its point of suspension is to the left of the midpoint. The weight is mg (*Figure 30*).

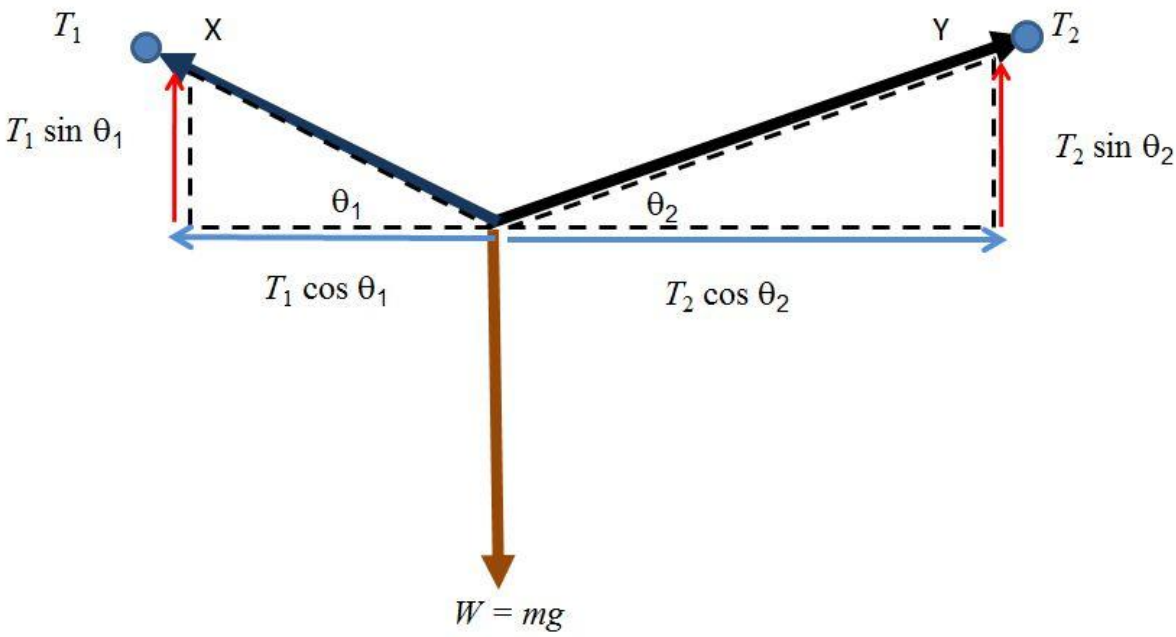


Figure 30 Triangle of forces that is not symmetrical.

We can apply these rules:

- The two horizontal components add up to zero.
- The two vertical components add up to the weight.

The vertical components:

$$T_1 \sin \theta_1 + T_2 \sin \theta_2 = mg$$

.... *Equation 9*

The horizontal components:

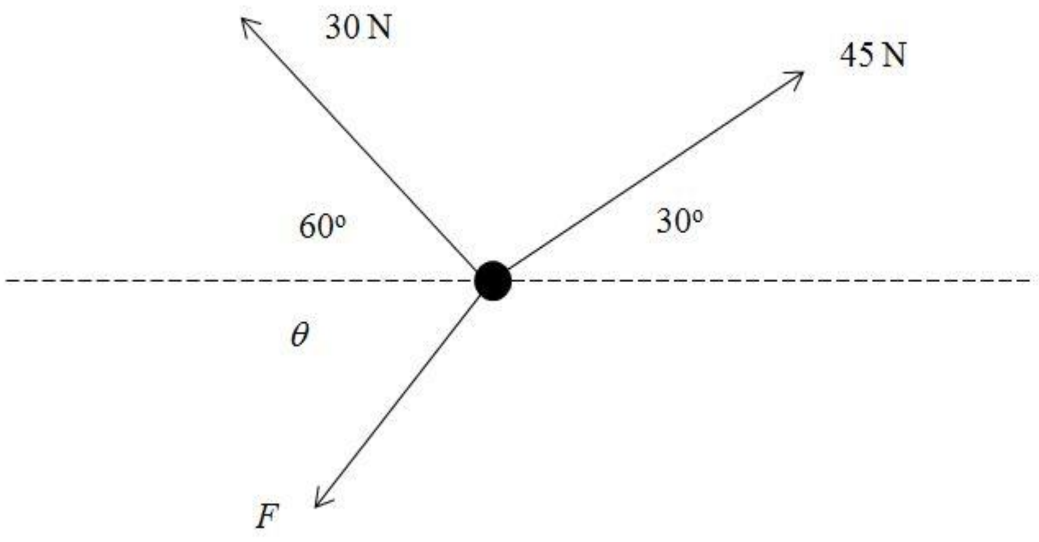
$$T_2 \cos \theta_2 + - T_1 \cos \theta_1 = 0$$

.... *Equation 10*

Since there are two unknowns, **simultaneous equations** may be needed.

Worked Example

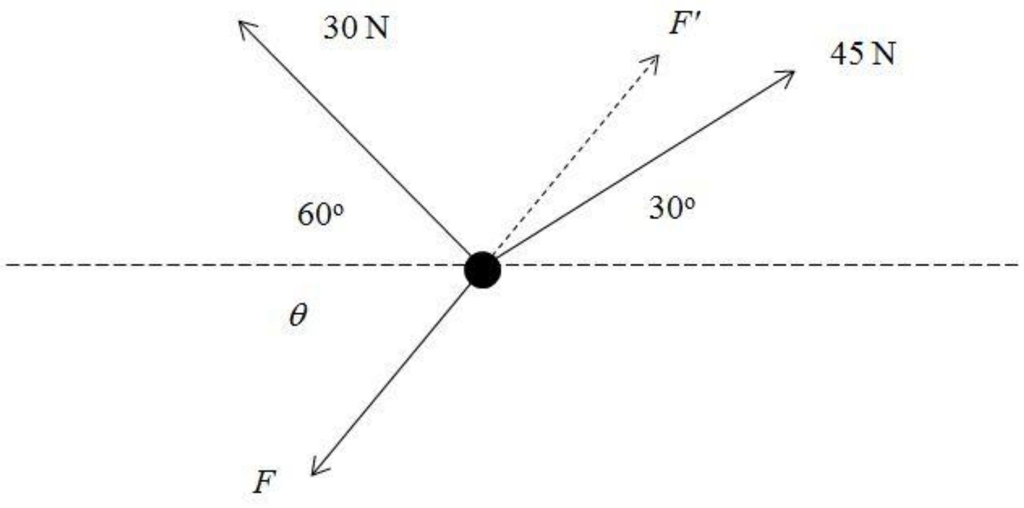
Three forces are shown below are in equilibrium. The diagram is NOT to scale.



Calculate the magnitude of the Force F and the angle θ.

Answer

This may appear quite difficult at first sight as there are two unknowns. The key thing to remember is that the resultant force is 0. Therefore, the force F is equal in magnitude and opposite in direction to the force F' (pronounced "eff-prime"). The force F' is the vector sum of the 30 N force, and the 45 N forces. Therefore, the solution to this is to resolve the vectors into vertical and horizontal components.



Resolve the 30 N and the 45 N forces into vertical and horizontal components. Upwards is **positive**.

Vertical Components are added:

$$\text{Vertical components} = 45 \sin 30 + 30 \sin 60 = 48.48 \text{ N upwards}$$

Horizontal components are added:

$$\text{Horizontal Components} = 45 \cos 30 + \ -30 \cos 60 = 23.97 \text{ N to the right}$$

The minus sign is there because the horizontal component of the 30 N force is from right to left, i.e. in the opposite direction to the horizontal component of the 45 N force.

So, we can work out the magnitude of the force F'

$$F'^2 = 48.48^2 + 23.97^2$$

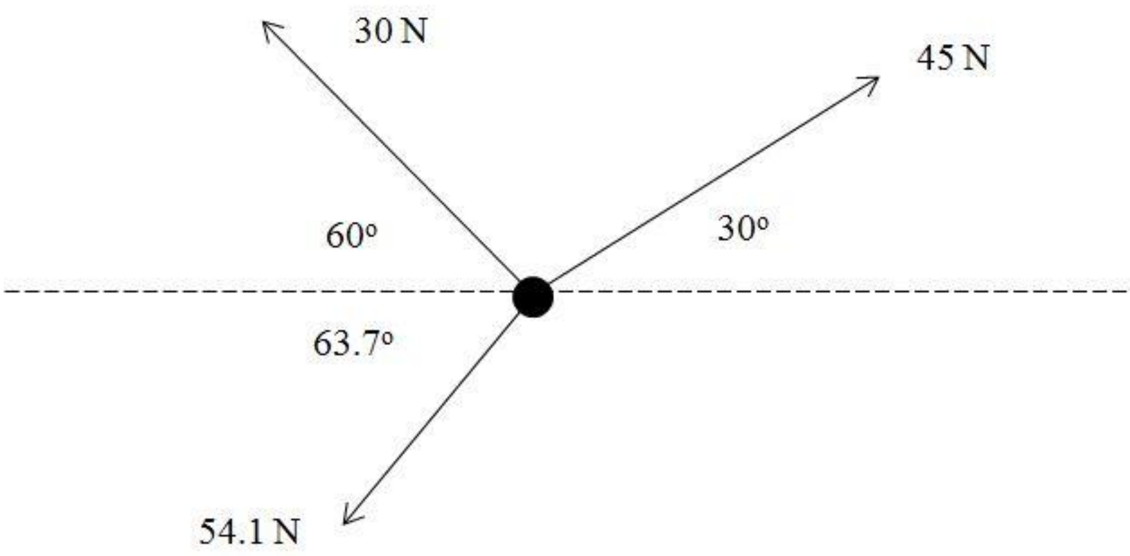
$$F' = 54.1 \text{ N}$$

We can work out the angle that the force makes with the horizontal:

$$\tan\theta = \frac{48.48}{23.97} = 2.023$$

$$\theta = \tan^{-1} 2.023 = 63.7°$$

So, our force in equilibrium looks like this:



Be careful that you don't assume that W is split evenly between $T_1 \sin \theta_1$ and $T_2 \sin \theta_2$. This is only true when the weight is half way between the ends.

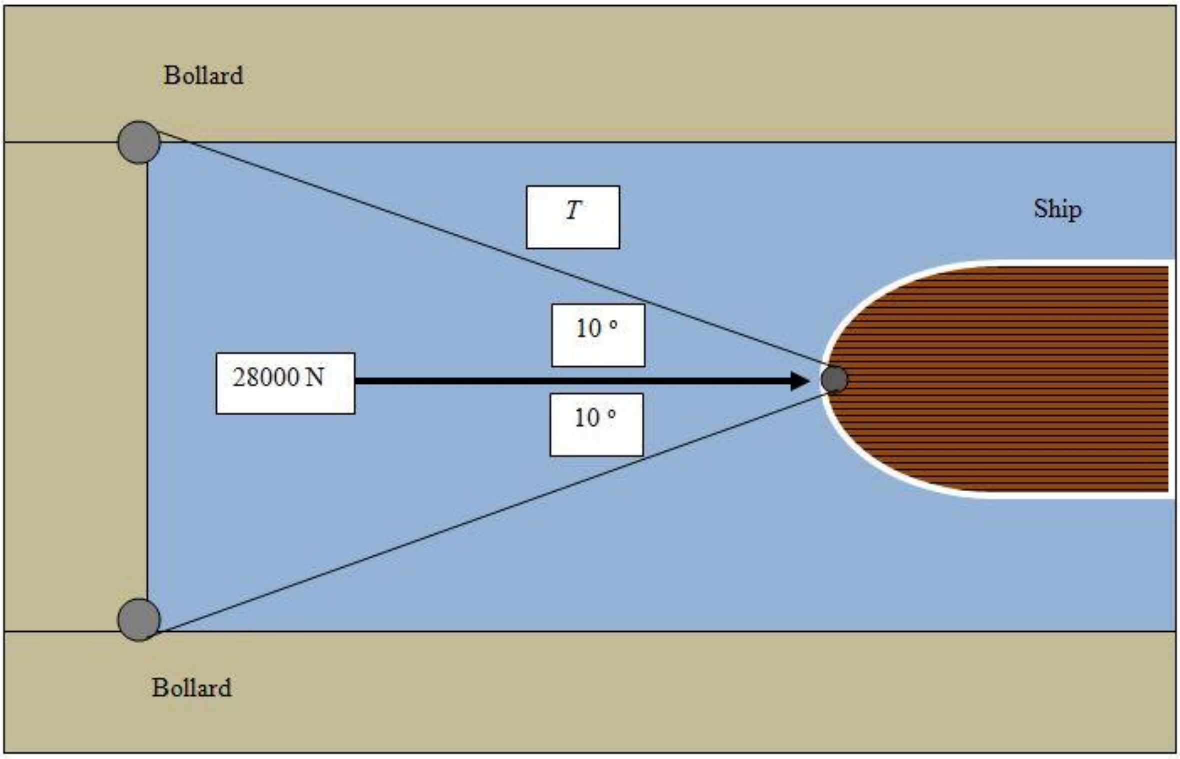
Many students write $T_1 \sin \theta = mg$ which is wrong.

Avoid using bearings (popular in Maths Mechanics) when calculating the angle between vectors - unless, of course, you are asked about planes in flight or ships on voyages.

Tutorial 5.02 Questions

5.02.1

A ship is in a dock and is secured to bollards on the harbour walls by two thick ropes. It is facing into a very strong wind which is putting a force of 28 000 N on the ship. This is shown in the diagram below:



(a) Calculate the tension T in the top rope.

(b) Calculate the force at 90 degrees to the wind that is acting on the bottom bollard. Which direction is it acting in?

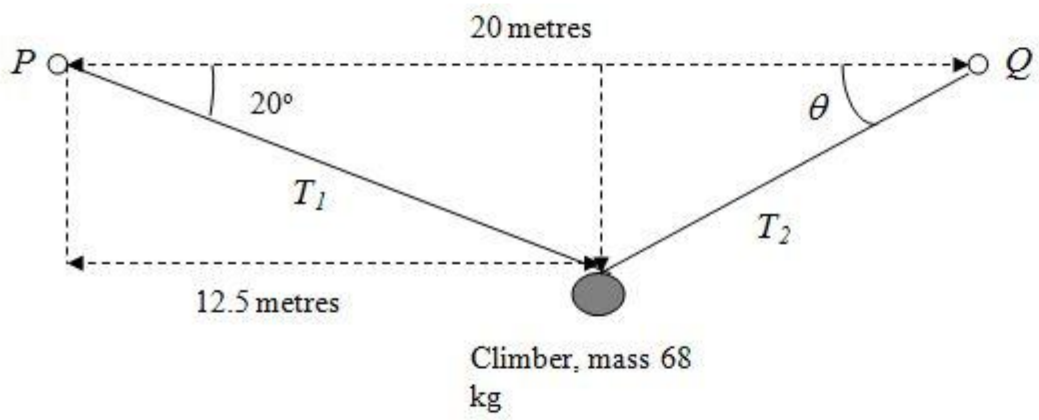
(c) What is the overall force acting on the ship? Explain your answer.

(d) The bottom bollard was not very well put in, and it gets ripped out of the harbour wall. Calculate the resultant force acting on the ship immediately after the bollard gives way and state the direction relative to the wind. 0° is parallel to the wind.

5.02.2 *(Challenge)*

A rock climber of mass 68 kg is making a traverse horizontally across a cliff face 20 metres wide between the fixed points *P* and *Q*. When he has got 12.5 metres across the rock face, he loses his footing and dangles in mid-air. While the climber is shouting at him to "get a life and do something useful", the companion, who is a keen physics student, sets out to measure the angle at *P* from the horizontal, which he finds to be 20° as in the diagram.

(Yes, I know, which rock-climber carries a protractor in his or her pocket?)



a. What is the weight of the climber?

b. Show that the angle θ is about 31°.

c. Show that force T_2 is $1.1 \times T_1$.

d. Calculate the values of forces T_1 and T_2.

Tutorial 5.03 Moments and Equilibrium

All Syllabi

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5.031 Mass and Weight

The **mass** of an object represents the **amount** of material in it. It is measured in **kilograms** and is a scalar. The mass results in **inertia**, the degree in which a body opposes the change in velocity. Mass does not change.

Weight is a **force** measured in **newtons**. It is the force that is the result of **gravity** acting on a force. Like all forces, weight is a vector, which always acts **vertically downwards**. Weight depends on the acceleration due to gravity.

Weight (N) = Mass (kg) × acceleration due to gravity (m s^{-2})

$$W = mg \quad \text{..............Equation 11}$$

The acceleration due to gravity on Earth is 9.81 m s^{-2}. It is also expressed as **force per unit mass**, 9.81 N kg^{-1}. On the Moon the acceleration due to gravity is 1.6 m s^{-2}.

5.032 Turning Effects of a Force

If we have hinged or pivoted body, any force applied changes the rotation of that body about the pivot. The turning effect is called a **moment** (*Figure 31*).

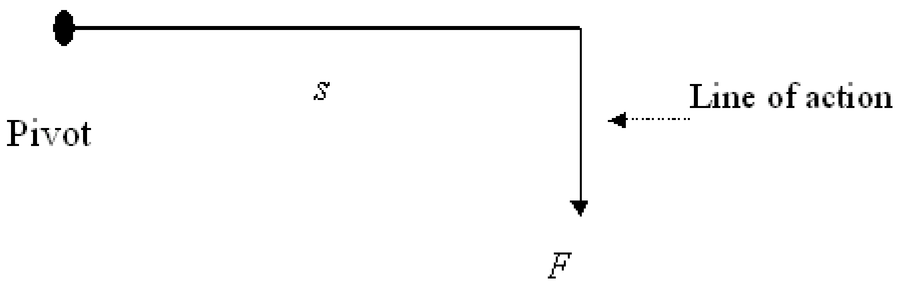

Figure 31 Moment of a force

The equation is:

Moment = force × **perpendicular** distance

In Physics Code:

$$\Gamma = Fs$$ .... *Equation 12*

This strange looking symbol, Γ, which looks like a gallows, is "Gamma", a Greek capital letter 'G'. Units are newton metres (N m). An alternative physics code for moment is τ, ("tau", a Greek lower-case letter 't').

Moments have a **direction**. As they are turning effects, we can talk of **clockwise** and **anticlockwise** moments. By convention, clockwise is **positive**.

Consider a trap door held by a piece of string, *BC*. ***P*** and ***Q*** are forces. The trap door is hinged about point *O*. (*Figure 32*)

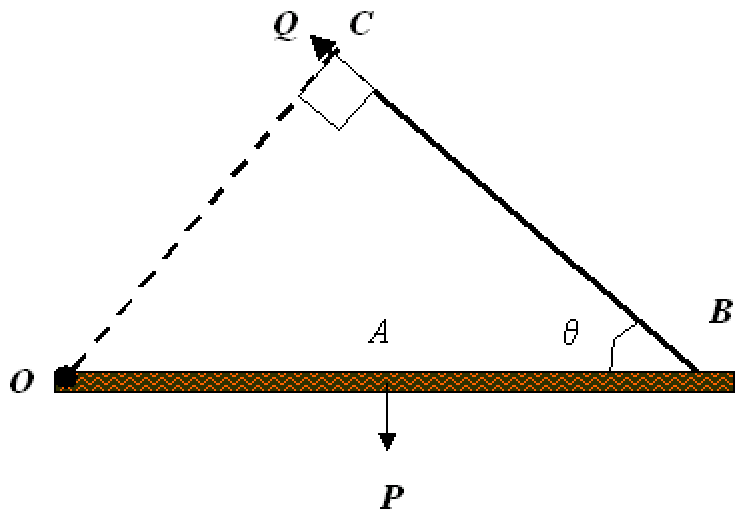

Figure 32 Moments about a trap door

The perpendicular distance of the line of action of force Q is the length of the line OC.

Moment of ***P*** about *O* = $P \times OA$

Moment of **Q** about *O* = $Q \times OC$

Note that the Moment of **Q** about *O* is [**Q** ×*OC*]. This is because *OC* is the **perpendicular** distance of the force **Q** from the hinge *O*.

If the trap door remains in equilibrium:

Anticlockwise moment (*Q* × *OC*) = clockwise moment (*P* ×*OA*)

This is the **Principle of Moments**.

Since $OC = OB \sin \theta$ we can say that

$$Q \times (OB \sin \theta) = P \times OA$$

5.033 The Principle of Moments

The principle of moments is an important rule that says:

- **The sum of clockwise moments about a pivot**... (1 mark)
- ...**is equal to the sum of anticlockwise moments**... (1 mark)
- ...**for a system in equilibrium**. (1 mark)

When asked to state the principle of moments, you must include these three points. A **pivot** is sometimes called a **fulcrum**.

Consider a beam of negligible mass, on which three forces in equilibrium are acting. The forces are arranged as in the diagram (*Figure 33*).

- F_1 is x m from the pivot.
- F_2 is y m from the pivot.
- F_3 is z m from the pivot.

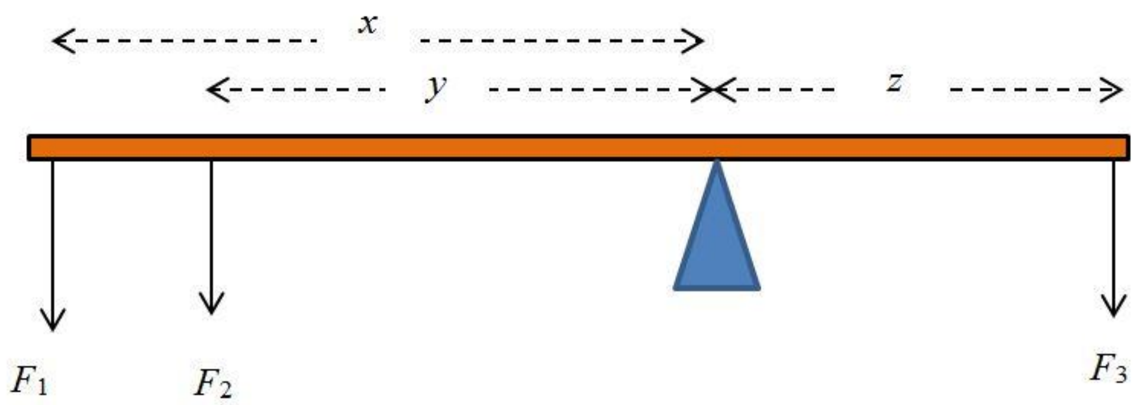

Figure 33 The Principle of Moments

Using the principle of moments we can write:

Clockwise moments = anticlockwise moments

Therefore:

$$F_3 z = F_1 x + F_2 y$$

.... *Equation 13*

5.034 Centre of Mass

We treat objects as **point masses** referring to a single point called the **centre of mass**.

In regular objects like a cube or a sphere, the centre of mass is in the middle. In some objects the centre of mass is outside the object (*Figure 34*).

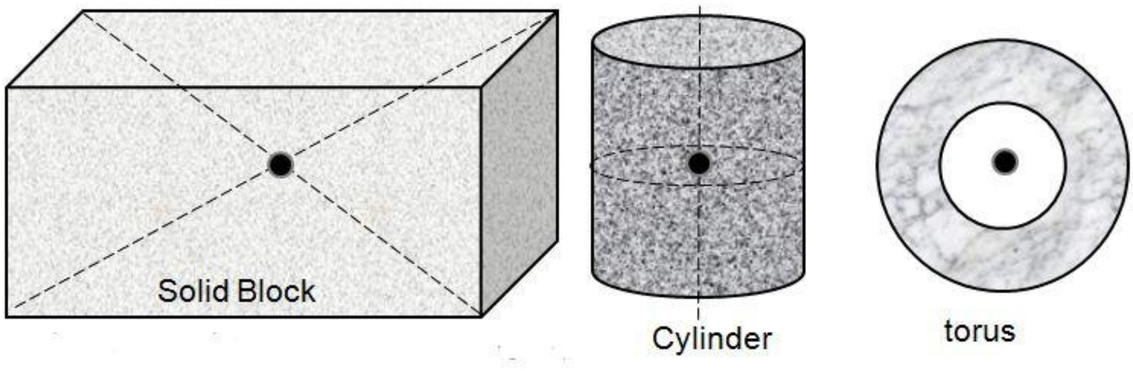

Figure 34 Centre of mass

The **centre of mass** (or **centre of gravity**) is the point through which the entire weight is said to act. Objects with a very low centre of gravity tend to be very stable. Some objects are so stable that they never fall over. Objects with a high centre of gravity are unstable.

5.035 Balancing

Many questions involve the balancing of see-saw around pivots. Let us look at some situations. In this case (*Figure 35*) the are two forces, A and B that are not equal. Therefore, to make sure that the bar remains horizontal, the distances x and y cannot equal.

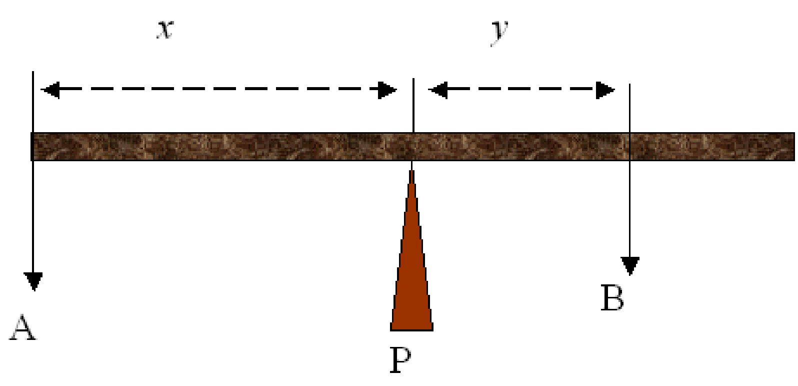

Figure 35 Principle of moments with different values of force, hence different distances.

This is the simplest case. The pivot is in the middle of a **uniform** bar. It means that the object is totally regular, and the **centre of mass** is in the **middle**. Therefore, we can ignore the mass of the bar. If the bar is balanced, we can say that:

anticlockwise moments = clockwise moments

$$Ax = By$$ .... *Equation 14*

Let us look at a case where we move the pivot P to one side. The centre of mass stays where it is and is NOT above the pivot. We now have to take it into account. Mass is not a force, but weight is. So, we need the weight, W (*Figure 36*)

$$W = mg$$

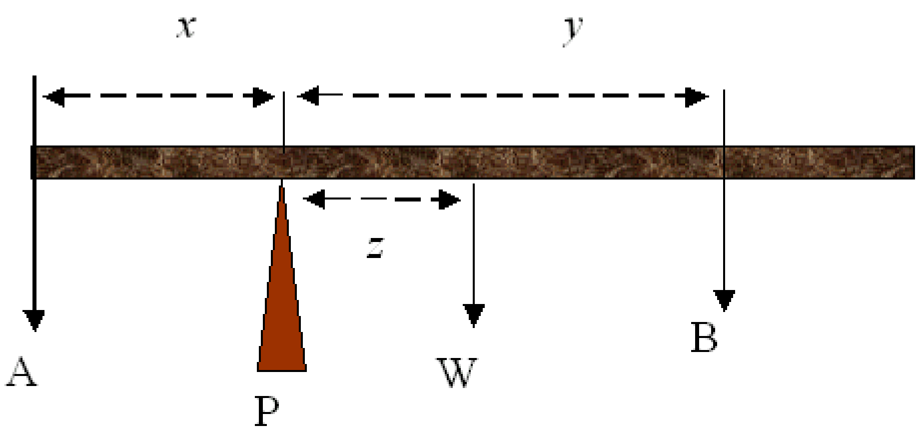

Figure 36 Equilibrium involving three forces

In this case, the **line of action of the weight**, W, is z metres from the pivot P. Applying the principle of moments we can say:

$$Ax = By + Wz$$ .... *Equation 15*

5.036 Couples

If two forces act about a hinge in **opposite** directions, there is an obvious turning effect called a **couple**. The resulting linear force from a couple is zero (*Figure 37*).

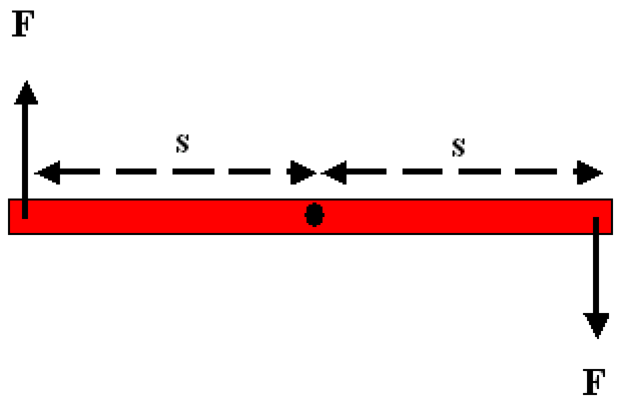

Figure 37 A couple

The couple is given by the simple formula:

$$\Gamma = 2\,Fs \quad \text{.................. Equation 16}$$

This strange looking symbol, Γ, is "gamma", a Greek capital letter 'G'. Couples are measured in **Newton metres** (Nm). The code τ (tau) is also used.

The turning effect is often called the **torque**. It is a common measurement made on motors and engines, alongside the power. Racing engines may be quite powerful but not have a large amount of torque. This is why it would not make sense for a racing car to be hitched to a caravan, any more so than trying to win a Formula 1 race in a 4 x 4.

5.037 Centre of Mass and Stability

A **stable** object does not tip over. Objects that have a high **stability** do not tip over easily. Their **centre of mass** is low down, near the base. This candlestick has a low centre of mass so that it does not tip over so easily.

The candlestick in *Figure 38*) is in **stable equilibrium**. If you push the top, it will drop back to where it was.

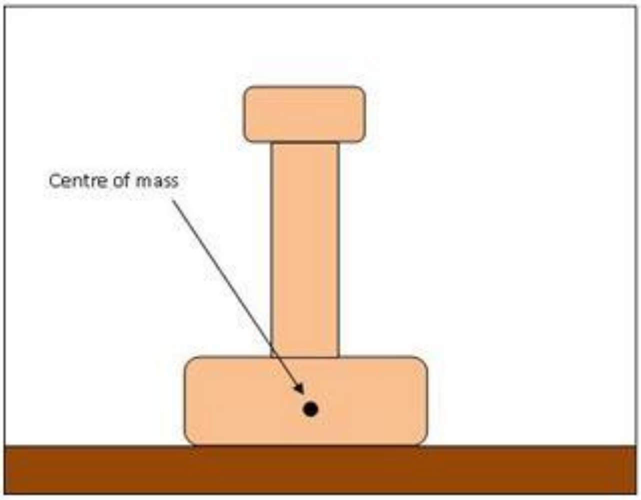

Figure 38 A candlestick in stable equilibrium

Now suppose we put it upside down (*Figure 39*). This time the centre of mass is high up. It is in **unstable equilibrium** and if you pushed the candlestick, it would tip over easily.

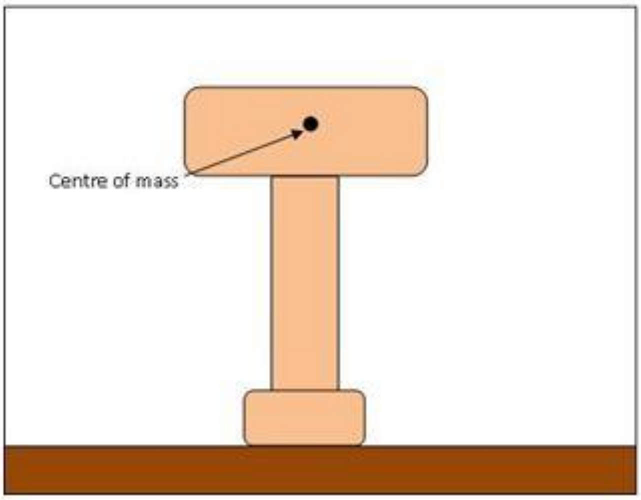

Figure 39 A candlestick in unstable equilibrium.

Now we put it on its side (*Figure 40*).

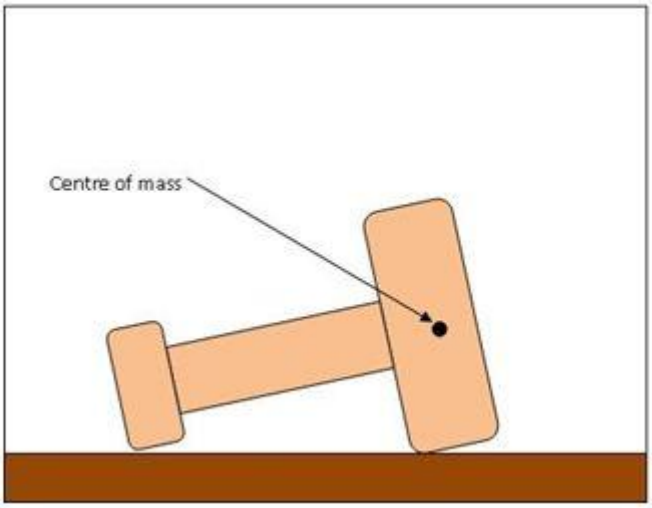

Figure 40 A candlestick in neutral equilibrium

If you push it, it will roll but will not tip over. It is in **neutral equilibrium**.

This wine rack uses the idea of moments to form a stable object (*Figure 41*).

Figure 41 This wine bottle stand is stable

5.038 Finding the centre of mass for irregular objects

We can find the **centre of mass** of an irregular object quite easily. If we let it hang freely, the centre of mass is directly below where we hang it from. We can find this line by hanging a **plumb line** from where we hang the object (*Figure 42*). In old text books, an object like this is called an **irregular lamina**. (The word *lamina* is a Latin word for leaf.)

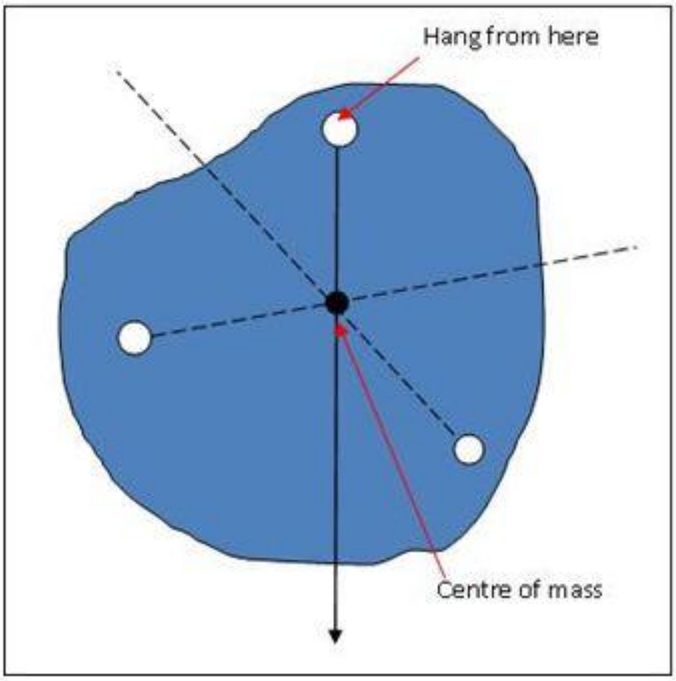

Figure 42 Finding the centre of mass for a flat irregular object.

We draw a line vertically downwards.

If we then hang the object from a couple of other points and draw the lines that go vertically downwards, the centre of mass is where the lines meet.

When the object is hanging freely, the centre of mass is **vertically** below the hanging point. The vertical arrow is called the **line of action of the weight.**

We will see the importance of stability in the next tutorial.

5.039 Some Examples using Moments

Sack Trolley

Now let's look at a sack trolley being pushed along a floor. The mass of the trolley and the load is m. The trolley is at an angle θ to the vertical. The distance from the centre of mass and the pivot is s, while the total length of the trolley is l. (*Figure 43*)

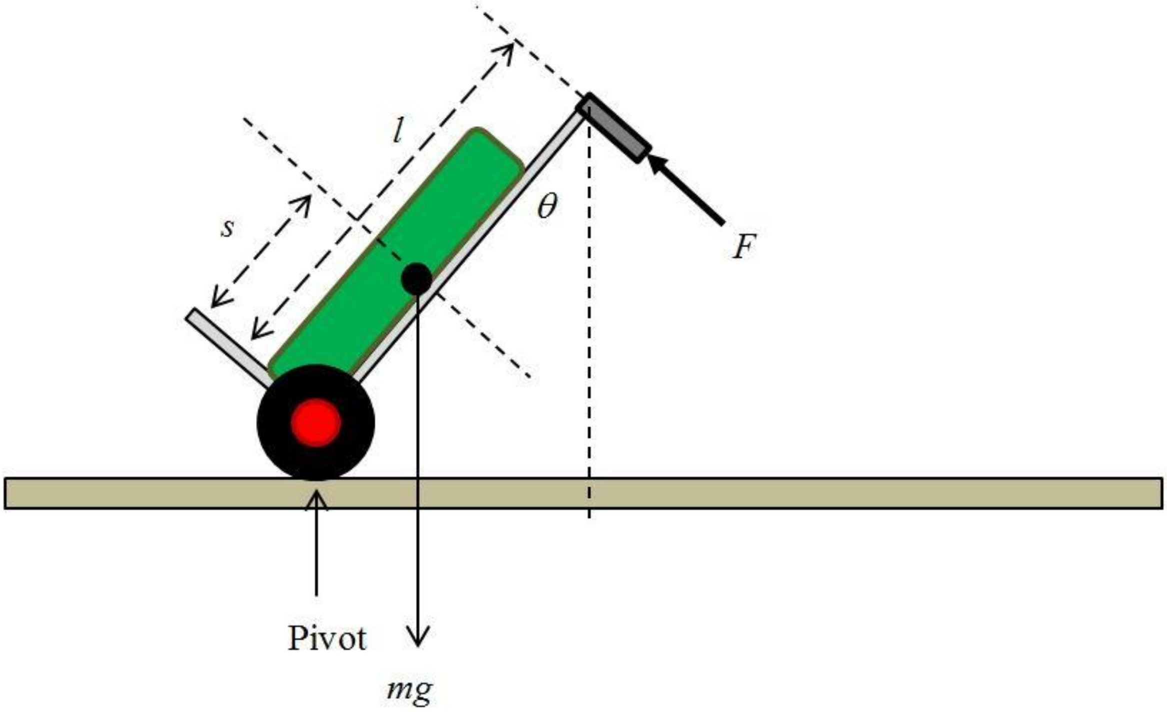

Figure 43 A loaded sack trolley

For the sake of simplicity, the centre of mass is being taken as both the centre of mass of the trolley and the load. We will take moments about the pivot.

The weight results in the clockwise moment:

$$\text{Clockwise moment} = mg \times s \sin\theta$$

.... *Equation 17*

The force F (the lifting force from the person pushing the trolley) gives the anticlockwise moment:

$$\text{Anticlockwise moment} = Fl$$

..............*Equation 18*

Ladder on a Wall

Consider a uniform ladder propped up against a wall between points A and B of mass m and length l propped up against a wall at an angle θ (*Figure 44*).

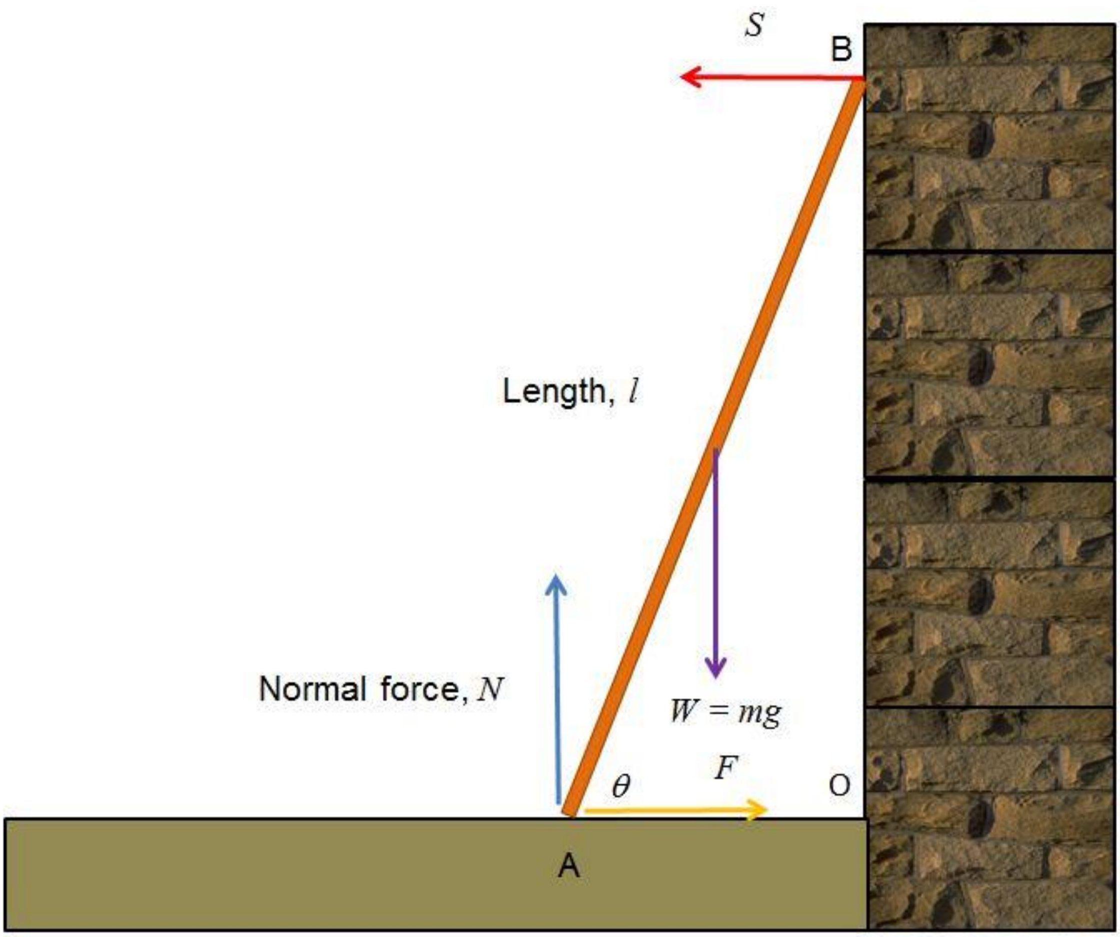

Figure 44 A ladder resting on a wall

There are four forces:

- The **weight**, W, which always acts vertically downwards from the centre of mass.
- The **normal force**, N, which acts at 90° to the ground.
- The **frictional force**, F, which acts horizontally towards the wall and stops the ladder from sliding outwards (which is not desirable).
- The **support force**, S, from the wall which acts in a horizontally from the wall.

A ladder can be treated using moments. Since it's uniform, the centre of mass is in the middle, i.e. half way along its length. The ladder is stable because all four forces form a closed rectangle (*Figure 45*):

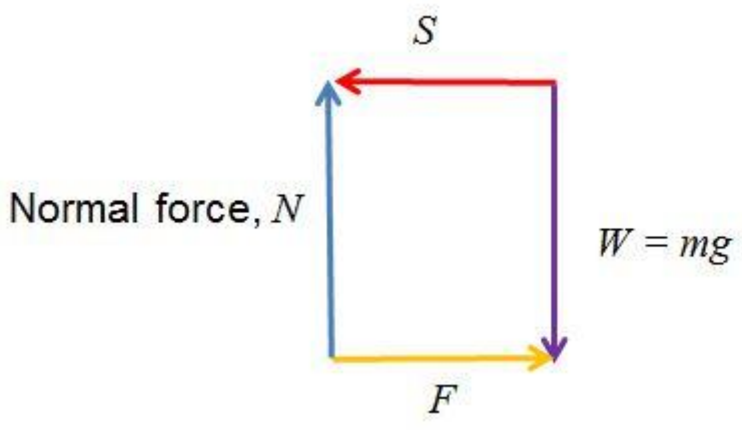

Figure 45 Rectangle of the four forces acting on a ladder propped up against a wall.

We can resolve vertically and horizontally:

$$N = W \text{ and } S = F$$

The weight of the ladder is mg.

The ladder is in contact with the ground at point A. Point A acts as the pivot. So, there is a **clockwise moment** acting about A which is given by:

$$\text{Clockwise Moment} = \frac{l}{2} mg \cos\theta$$

.... *Equation 19*

The force, S gives out an **anticlockwise** moment. Now we can work out the anticlockwise moment about A:

$$\text{Anticlockwise moment} = Sl \sin\theta$$

.... *Equation 20*

This is because the moment is the force S multiplied by the **perpendicular distance** from the pivot (*Figure 46*).

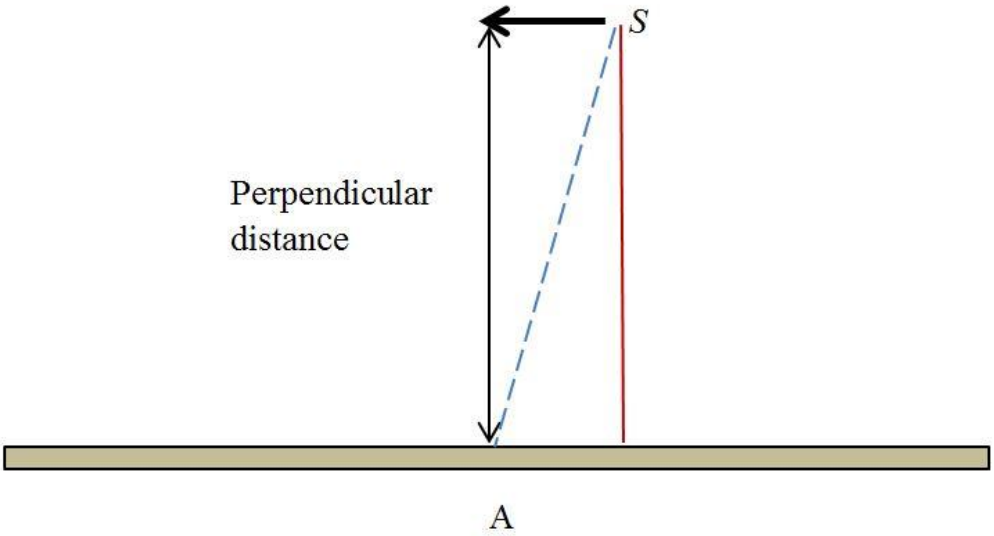

Figure 46 The moment provided by force S.

By the principle of moments, we can write:

$$Sl\sin\theta = \frac{l}{2}mg\cos\theta$$

.... *Equation 21*

which tidies up to give and rearranges to give:

$$S = \frac{mg\cos\theta}{2\sin\theta}$$

.... *Equation 22*

Notice that the l terms cancel out. Therefore, the support force of the wall on the ladder, S, that acts **away** from the wall, is **independent** of the length. The frictional force, F, has the same value as S, but acts **towards** the wall.

Tutorial 5.03 Questions

5.03.1

The spanner in the picture (*Figure 47*) below is 30 cm long and the nut in question has to be tightened to a torque (moment) of 85 N m. What force must the fitter apply?

Figure 47 Spanner turning a nut

5.03.2

The trap door in the diagram (*Figure 48*) has a mass of 12 kg.

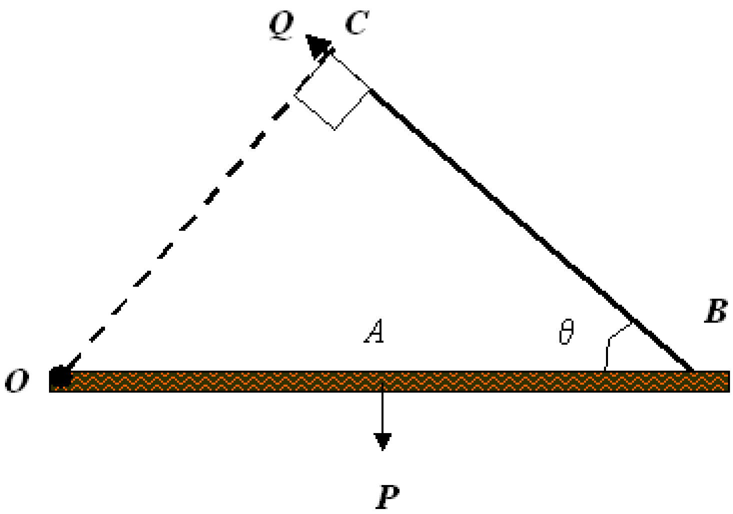

Figure 48 A trap door in equilibrium

The centre of mass is at A. The weight is Force P. What is the value of force P?

The angle θ is 25 degrees. The trap door is 100 cm wide. What is the value of force Q?

(Use g = 9.8 m s^{-2})

5.03.3

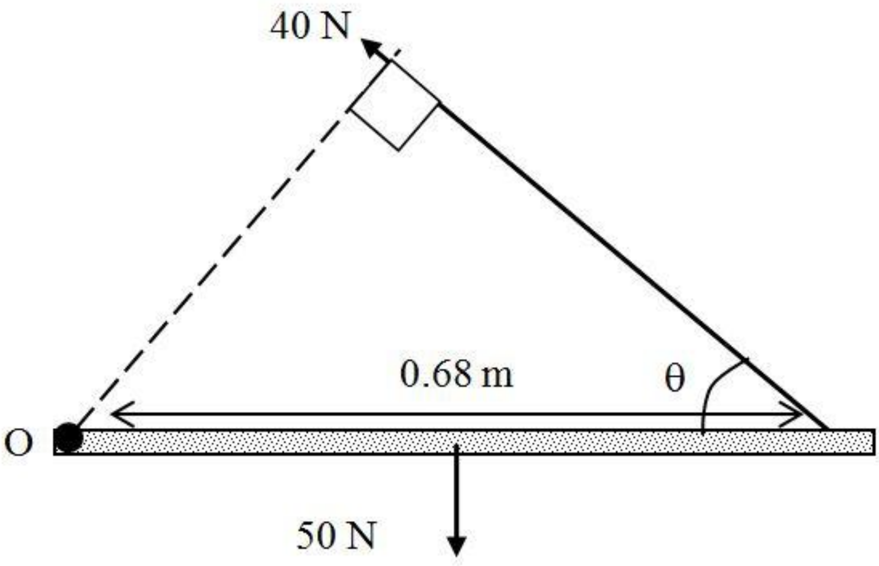

Figure 49 Trap door problem

Use the quantities in the diagram above to calculate the angle θ.

5.03.4

Some children are playing on a seesaw as shown in the diagram (*Figure 50*). The see-saw is a plank of wood 3.0 m long, with a pivot exactly in the middle. The centre of mass is directly above the pivot, so we can ignore it.

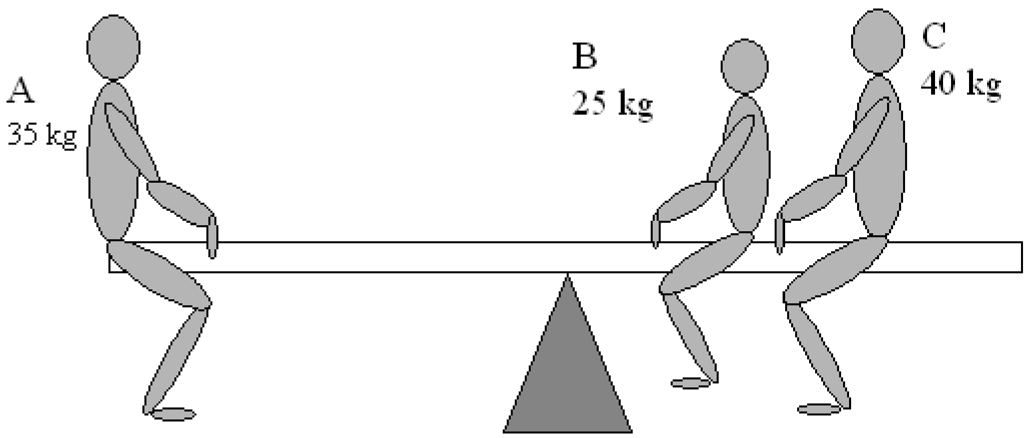

Figure 50 Children on a seesaw

(a) What are the weights of the children?

(b) Child B is sitting 0.4 m from the pivot. Where should child C sit so that the see-saw remains level?

(c) Child C misses its footing and falls off the end. What will happen to the others?

Use $g = 9.8 \text{ m s}^{-2}$

5.03.5

Figure 51 A novelty wine bottle stand uses moments

Look at the picture above.

(a) Where is the centre of mass in the bottle?

(b) Where is the pivot?

(c) Where does the line of action from the centre of mass of the bottle act?

(d) Use the principle of moments to explain how this system balances.

5.03.6

A trolley with its load has a combined mass of 50.0 kg. The total length of the trolley is 1.10 m, and the centre of mass is 30.5 cm from the wheels. The trolley is being held at an angle of 40° to the vertical. Calculate the force that is needed to hold it at that angle.

(Use g = 9.81 N kg^{-1})

5.03.7

A ladder of mass 20 kg and 3.0 m in length is propped up against a wall at an angle of 72°.

Calculate the support force on the wall.

Which data item is irrelevant?

g = 9.81 N kg^{-1}.

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5.041 Stability in Vehicles	5.042 Stability in Aeroplanes

Moments can be used to explain stability (or otherwise) in motor vehicles.

5.041 Stability in vehicles

Some people like to buy **sports utility vehicles** (SUVs). They are very fashionable as luxury cars (*Figure 52*).

Figure 52 Sports utility vehicle

They remind me of a computer I used at work more years ago than I care to remember. It was called a *Televideo* but that was a misleading name. It was neither a telly nor a video. In the same way, an SUV is neither a sports car, nor a utility vehicle. A **sports car** is designed to have nimble and agile handling characteristics, but many of these lumpish and brutish vehicles have dreadful handling characteristics that make them rather unpleasant to drive.

Utility vehicles are designed to lug large and possibly dirty objects about the country. However, a good number of SUV owners would have a fit at having dusty bags of cement in the back of their exceedingly expensive vehicles with their even more expensive personalised number plates. Some may tolerate muddy dogs in the rear luggage compartment, especially if they have spent a satisfying afternoon blasting woodpigeons from the sky…

Some of these hideous hulks luxury cars have serious stability problems when going round sharp corners. When you have spent £/€70000 on such a vehicle, you don't want it rolling over…

Let us look at how we can explain **stability** in vehicles. This bus has a low centre of mass and a wide **track** (distance between the wheels). You can see that there is a **line of action** of the weight that acts vertically downwards from the centre of mass (*Figure 53*).

If we tip the bus over:

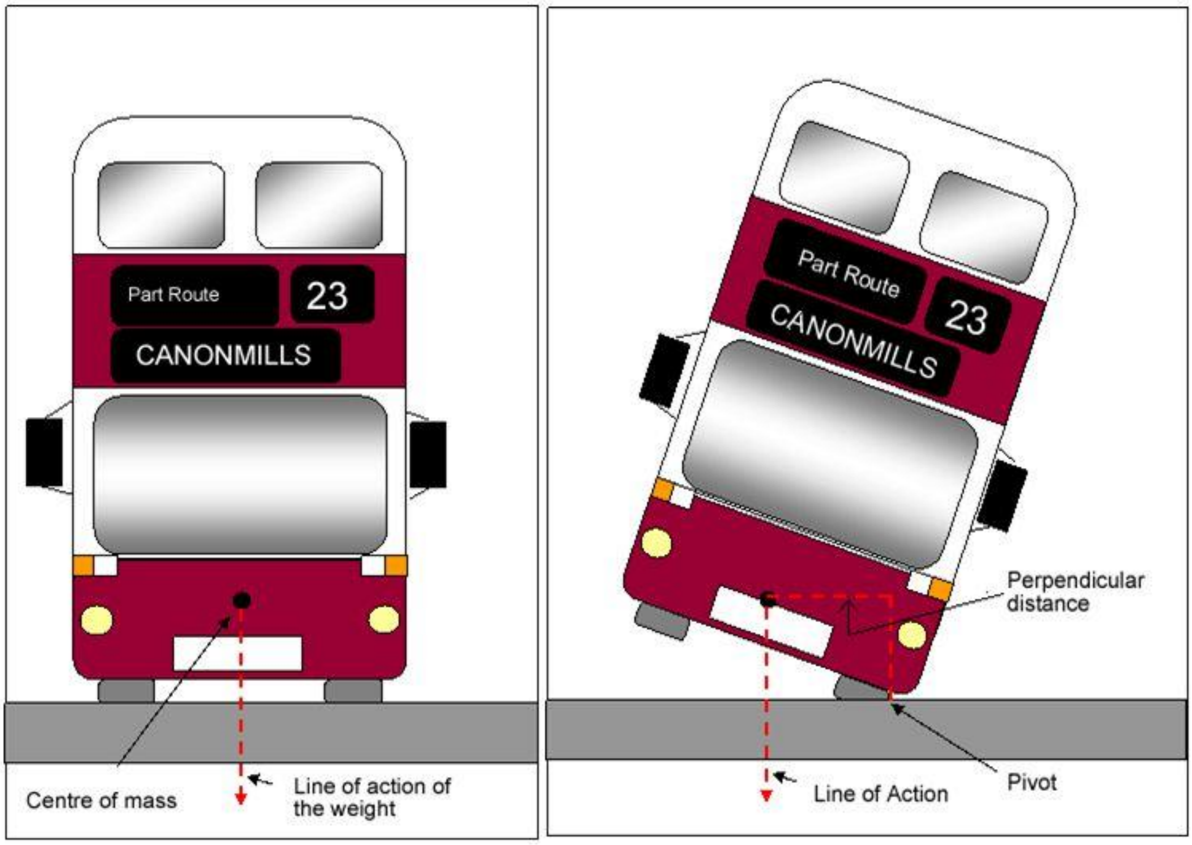

Figure 53 Tilting a bus.

The line of action of the weight is still acting vertically downwards, but one of the tyres is acting as a pivot. There is an **overall turning moment**; in this case it's anticlockwise, so the bus will go back to the vertical. Let's analyse this further (*Figure 54*).

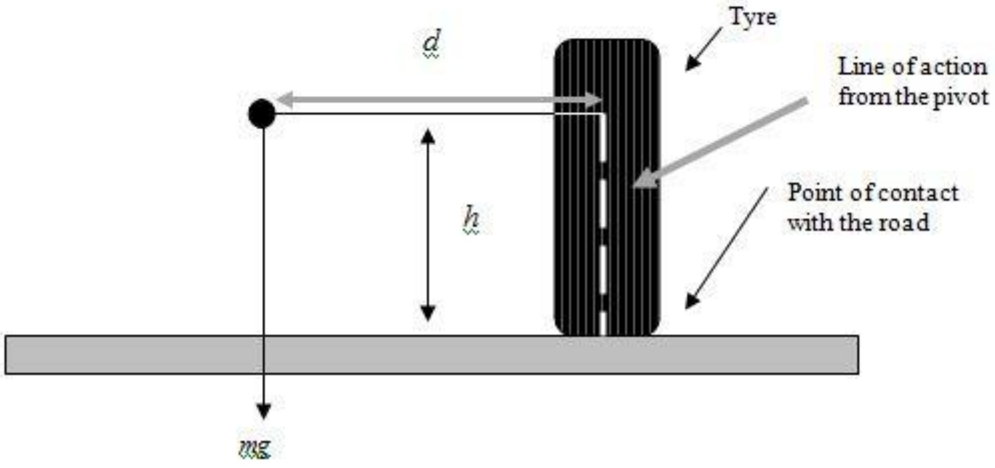

Figure 54 Force acting on a single vehicle wheel.

The bus has its centre of mass half-way between the wheels. The distance from the centre of mass to the tyre is d metres, and its height above the road is h metres. If the mass of the bus is m kilograms, its weight is mg newtons.

Suppose we tip the bus over clockwise by an angle of θ to the road (*Figure 55*).

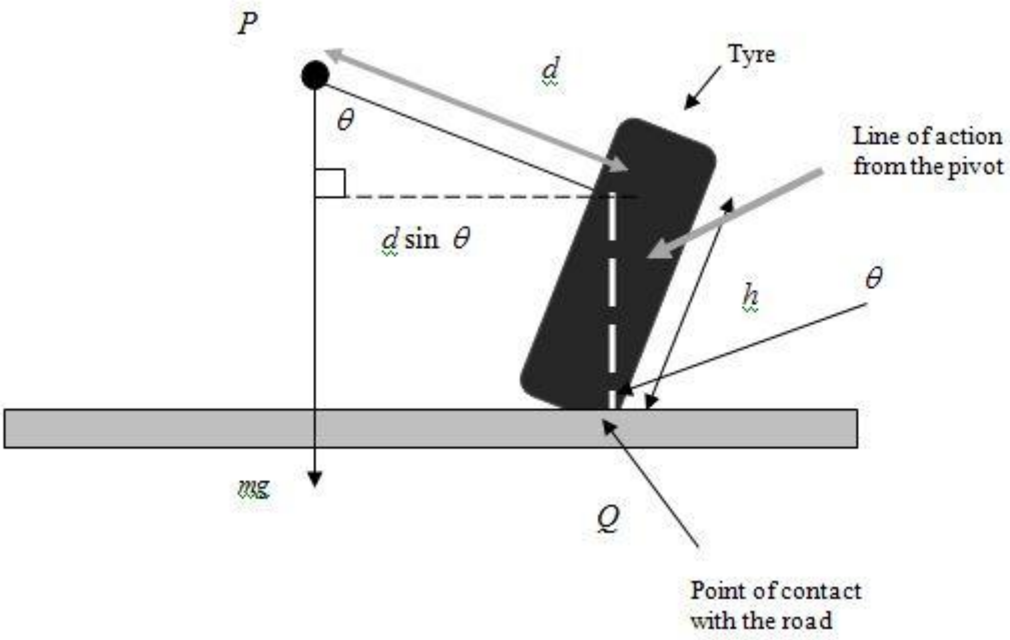

Figure 55 Tilting a vehicle

The angle the line of action from Q is θ. By simple geometry we can say that angle P is θ as well.

There will be a turning moment anticlockwise. This is given by:

$$\text{Moment} = mg \times d \sin \theta \quad \text{............ Equation 23}$$

The bus will go back onto its wheels (with a bang).

There is a **critical** point at which the bus might fall back or tip over. This is where the centre of mass is directly above the point of contact of the tyre with the road. Point P is **vertically** above point Q (*Figure 56*).

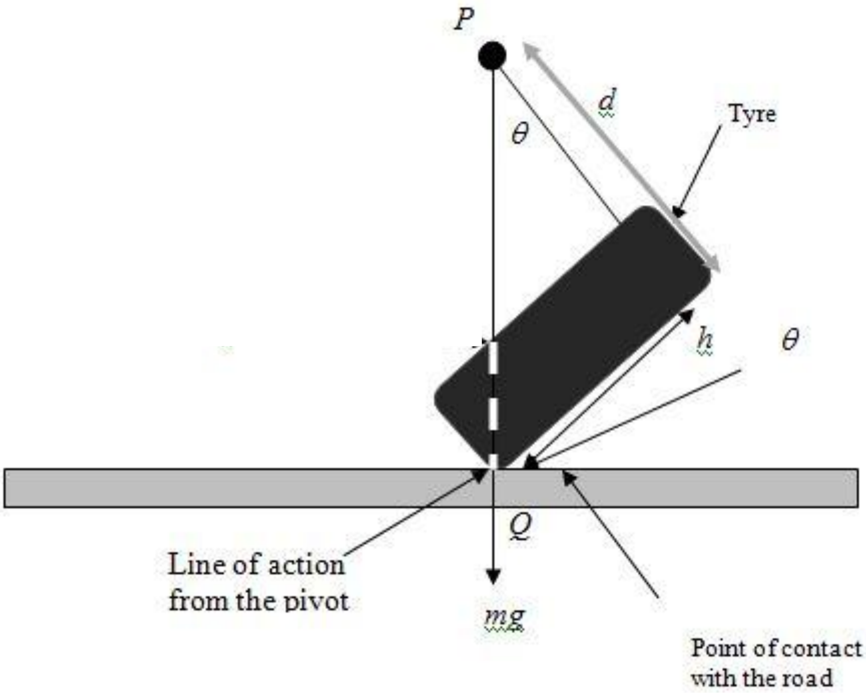

Figure 56 The critical point at which a vehicle will tip over.

If we know what distance d is and the height h is, we can easily work out the angle at which the bus will tip over. The distances are shown in the diagram (*Figure 57*).

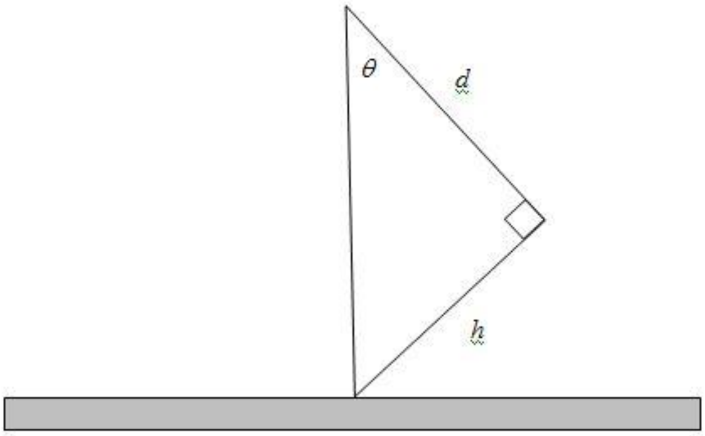

Figure 57 Simplified version of Figure 56

Therefore:

$$\tan\ \theta = h/d$$ .... *Equation 24*

If the bus tips over θ degrees to the horizontal, it is also tilting at θ degrees to the vertical.

Now suppose the bus tips further (*Figure 58*):

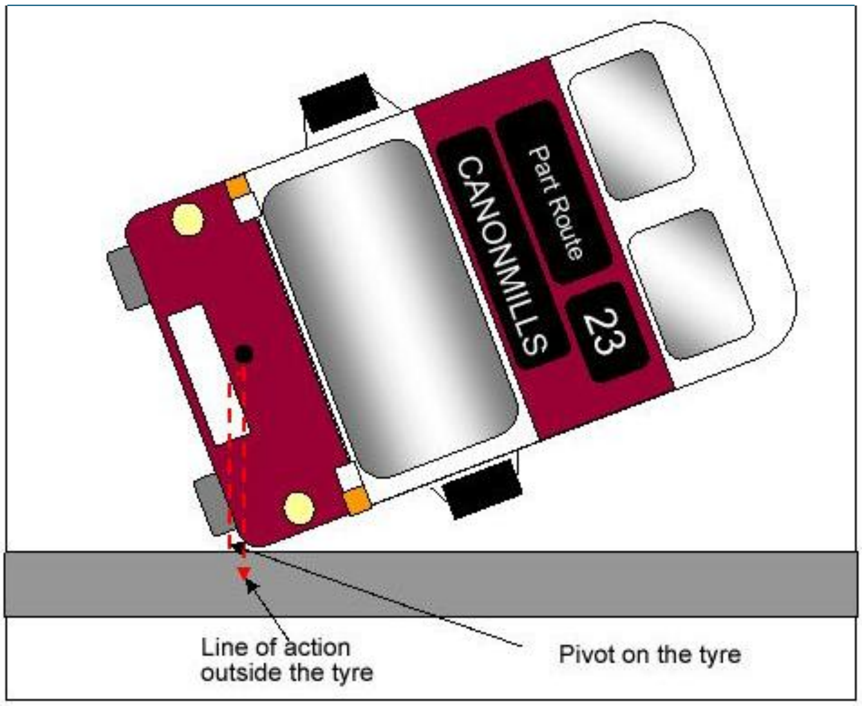

Figure 58 This bus is tipping over onto its side.

This time the line of action of the weight is to the outside of the tyre, so the turning moment is clockwise. The bus tips over on its side. (This has happened; a driver was late going off shift and was hurrying to get back to the garage. Going too fast round a sharp bend, the bus tilted too much and tipped onto its side.)

The line of action of the force is now outside the wheel, so there will be a **clockwise moment** to pull the bus right over (*Figure 59*).

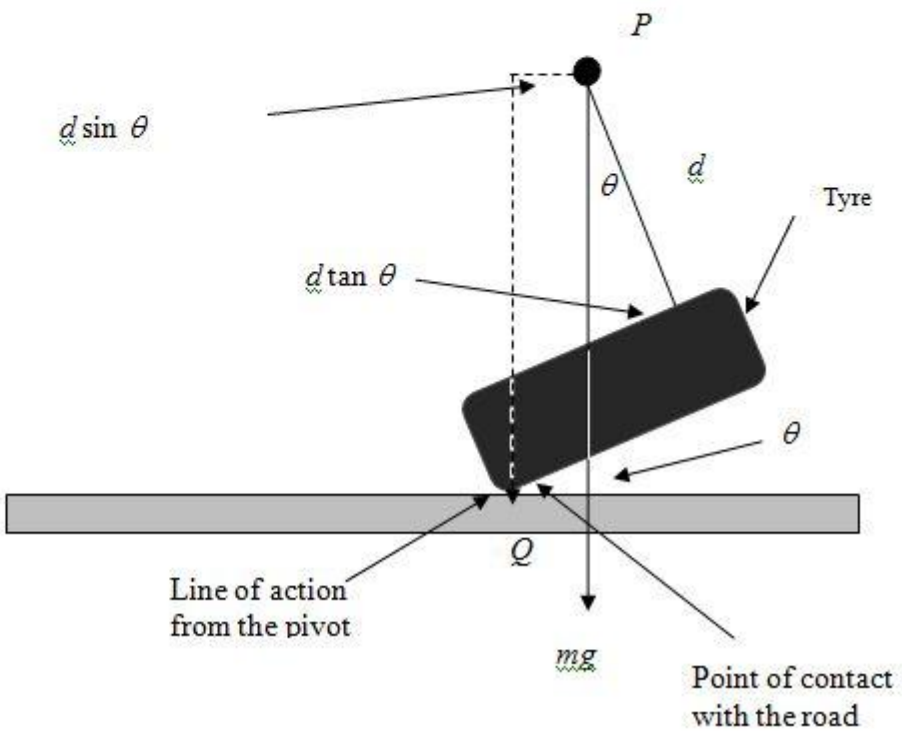

Figure 59 The weight provides a clockwise moment to tip the bus over.

The turning moment will be given by:

$$\text{Moment} = mg \times d \sin \theta$$ .... *Equation 25*

Although they are tall, double-decker buses are very stable. They test buses by putting lots of sandbags on the seats upstairs (with nothing downstairs) and tilt them over on a tilting platform. The centre of mass is low enough to ensure that they are tilted to more than 60° off the vertical before they tip over.

Lorries have a higher centre of mass on their trailers, due to the load. If you live in the country and get stuck behind a hay-lorry, you may see it swaying alarmingly. This kind of accident tends to happen with lorries when they drive through strong crosswinds on exposed roads.

5.042 Stability in Aeroplanes

When an aeroplane flies, there is a **centre of lift**, which is an imaginary point through which all the lift from the wings appears to act. Ideally the **centre of mass** of the aeroplane should be directly underneath this. Since the line of actions of both forces coincide, there is no turning moment, and the aeroplane is stable (*Figure 60*).

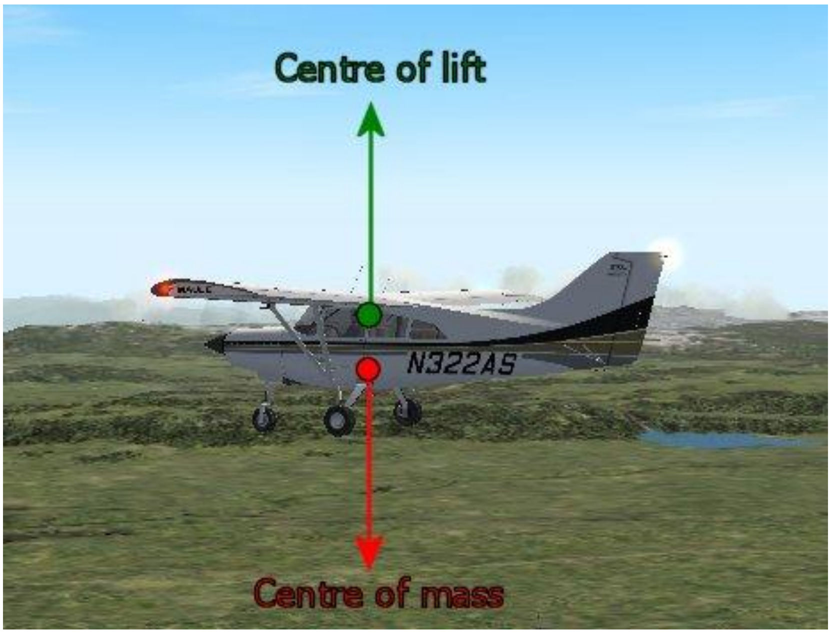

Figure 60 Force of lift and mass acting on a light plane

When the pilot uses the **elevators** (flaps at the tail to make the aeroplane move up and down), a force is applied to the tail, causing a turning moment to act (*Figure 61*).

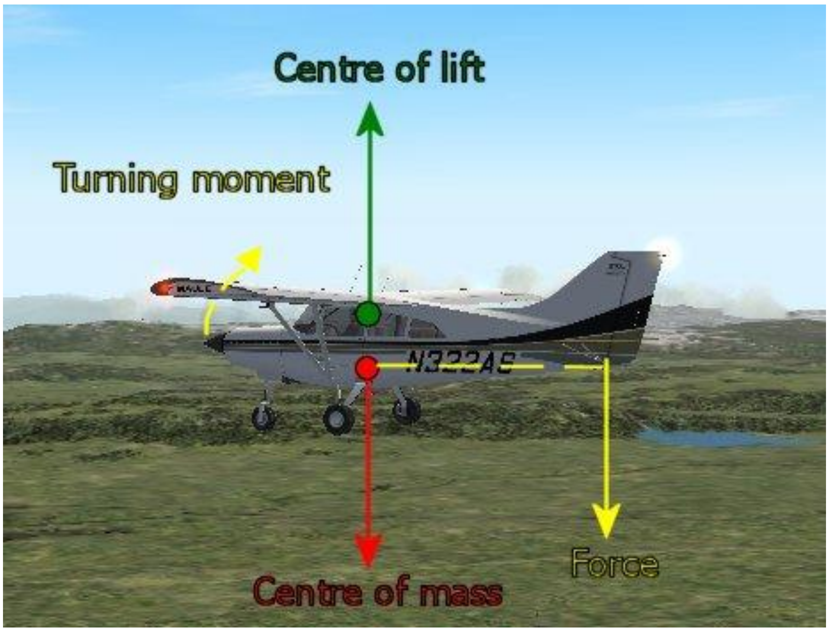

Figure 61 A downwards force on the tail causes a clockwise turning moment.

When the pilot wants to turn, he (or she) uses the **ailerons** (flaps at the end of each wing). One aileron is raised, and the other is lowered, to make a *couple*. The aeroplane rolls (*Figure 62*).

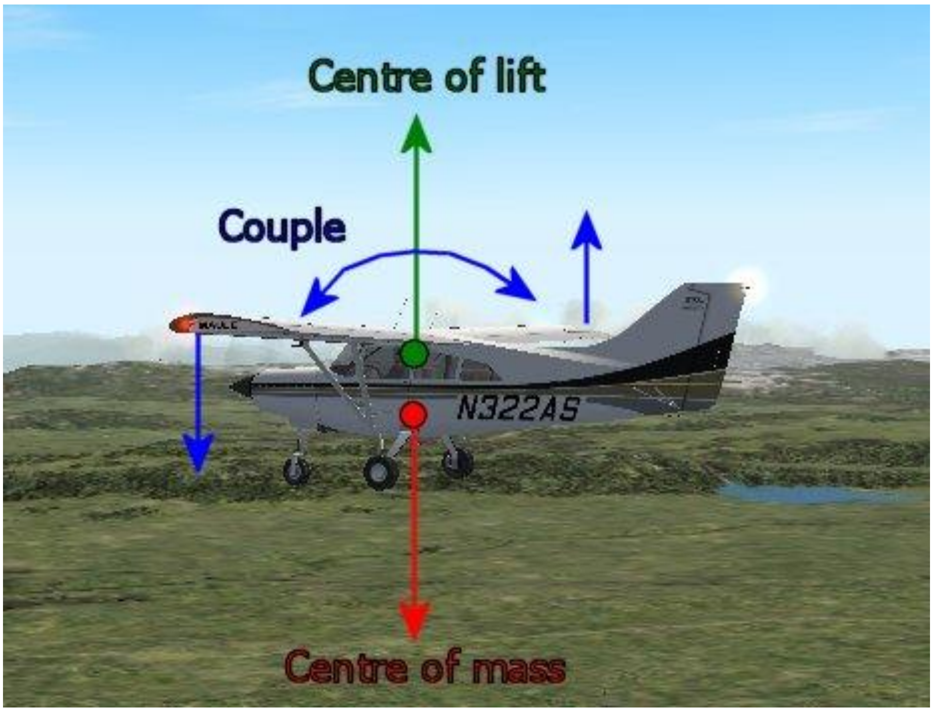

Figure 62 The ailerons make a couple to roll the aeroplane and makes it turn.

Now suppose the aeroplane takes off with both wing tanks full of petrol. The pilot can take petrol from both tanks, and he selects which tank to use. He needs to keep both tanks balanced, by using 10 litres from one tank, then 10 litres from the other tank. Every few minutes he needs to change over the tanks.

Suppose he forgets to change over (easily done) and takes almost all the fuel from the right tank. Question 5.04.3 gets you to think about this.

The management of the fuel is an important part of the air-work of the pilot of a light aeroplane. However, many light aeroplanes can take petrol from both tanks at the same time. Unless there is a problem with one of the tanks, that is what is done normally.

If the aeroplane is not balanced, it is possible for the turning moment to be cancelled out by using the control surfaces. All aeroplanes have trim tabs which are little flaps that can push the nose up or down to balance the plane.

There was a tragic accident involving an aeroplane in Africa. One of the passengers had smuggled on board a young crocodile that was a pet. It got out from its box and all the passengers rushed to the front of the plane. There was a big nose-down turning moment which the pilot could not balance with his controls. The plane nose-dived into the ground.

Tutorial 5.04 Questions

5.04.1

If the height h was the same as d, what is the maximum angle of tilt that the bus could make?

5.04.2

A lorry has a mass of 10 000 kg. Its track (width between the wheels) is 2.0 m, and its centre of mass is 0.75 m above the road surface.

It is travelling due North. A crosswind from due West is acting on the side of the lorry. This is acting on the lorry with a force of 5000 N, and the line of action of the force is 3.0 m above the road surface.

(a) Show that the maximum angle to the vertical that the lorry could tilt before it tips over is about 53°.

(b) Calculate the moment made by the wind, hence the angle to which the lorry would tilt.

(c) Will the lorry tip over?

Use g = 9.8 N kg^{-1}

5.04.3

Now suppose the aeroplane takes off with both wing tanks full of petrol. The pilot can take petrol from both tanks, and he selects which tank to use. He needs to keep both tanks balanced, by using 10 litres from one tank, then 10 litres from the other tank. Every few minutes he needs to change over the tanks.

Suppose he forgets to change over and takes almost all the fuel from the right tank.

How do you think this will affect the handling of the plane? Explain your answer.

Tutorial 5.05 Moments and Bridges	
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5.051 Bridges

At its simplest, a **bridge** is a plank of wood placed between two supports (often called **abutments**). As the plank of wood has a mass, it must also have a centre of mass. At this level, we will assume that the plank is totally regular, and its centre of mass is exactly in the middle (*Figure 63*).

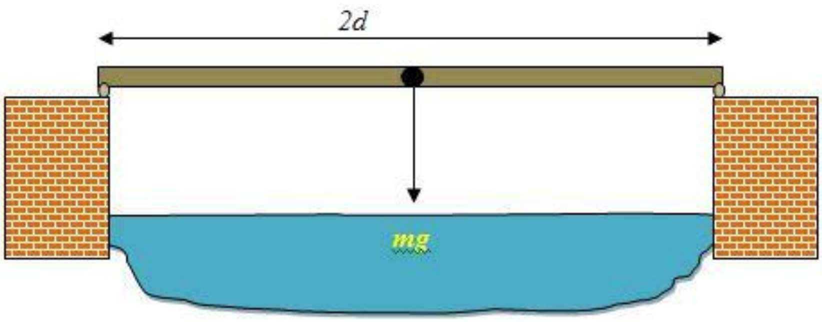


Figure 63 A simple bridge

Notice that at each end there is a pivot, around which we will need to take moments. Even if there were no actual pivot mechanism, we treat the bridge as if there were. If the plank broke in the middle, the right-hand half would turn about the right pivot, and the left half around the left pivot (*Figure 64*).

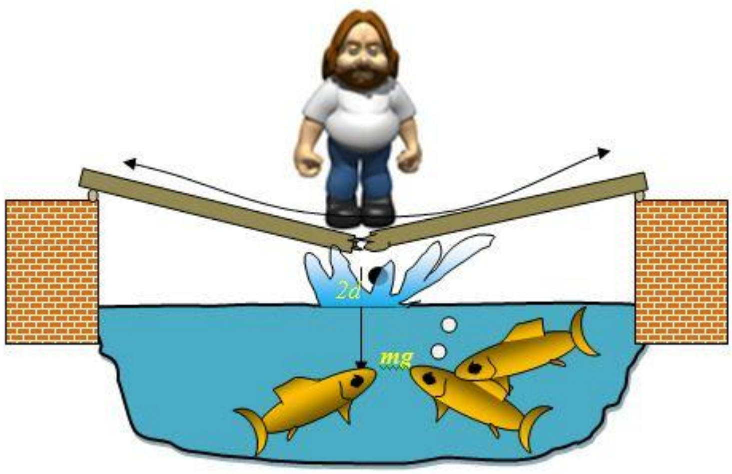


Figure 64 A bridge breaking in the middle

5.052 Moments and Bridges

So let us have a look at the moments around each pivot (*Figure 65*)

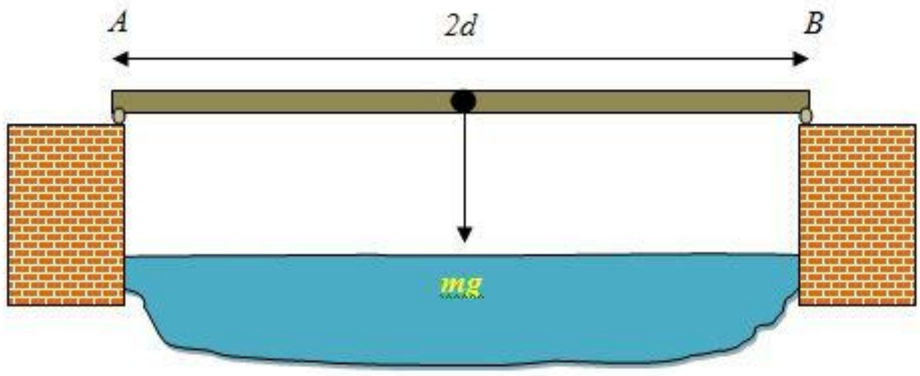


Figure 65 Moments about the abutments of a bridge.

$$\text{Clockwise Moment} = mg \times d$$ .... *Equation 26*

$$\text{Anticlockwise moment} = mg \times d$$ .... *Equation 27*

Since force = moment ÷ distance, it doesn't take a genius to see that the force on each pivot, A and *B*, is given *by*:

$$F = mg/2$$ .... *Equation 28*

When we look at forces acting on a bridge with traffic on, we are usually asked to consider the force acting on each **abutment** (the pier at each end). We need to take moments about each end. Bridge engineers have to do this carefully, otherwise there would be a disaster. We will look at this by a worked example.

Now let's put a load on the bridge of mass m_1 kg, x metres from one end of the bridge (*Figure 66*).

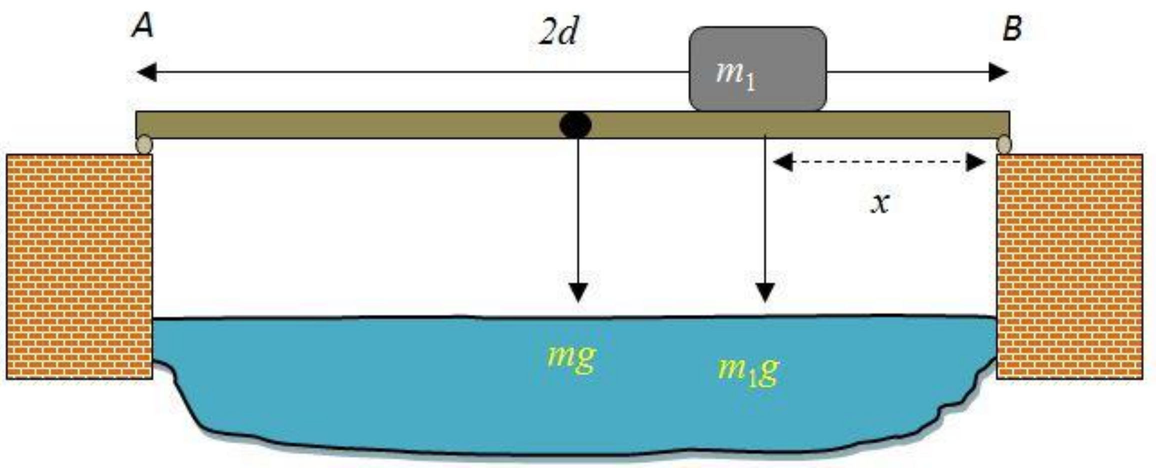


Figure 66 Putting a load on the bridge.

The centre of mass is in the same place, but we now have an extra load of m_1g on the bridge. Taking moments about B:

$$\text{Moment} = (mg \times d) + (m_1g \times x)$$

Taking moments about A:

$$\text{Moment} = (mg \times d) + (m_1g \times [2d - x])$$

Force on A can be worked out:

$$\text{Force on } A = \text{Moment about } B \div 2d$$

$$= \frac{(mg \times d) + (m_1g \times x)}{2d}$$

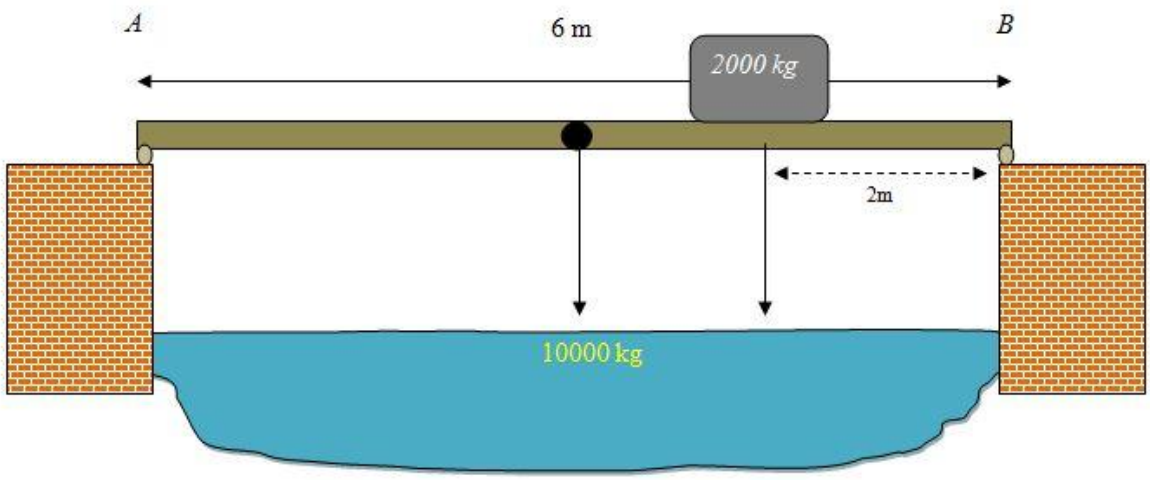
Similarly, the force on B can be worked out:

$$\text{Force on } B = \text{moment about } A \div 2d$$

$$= \frac{(mg \times d) + (m_1g \times [2d - x])}{2d}$$

Worked Example

A bridge has a mass of 10 000 kg and is made of a uniform beam *AB* 6.0 m long as shown. A 2000 kg mass is place on the beam 2.0 m from the end *B* as shown



What is the force acting on each end of the bridge?

Use g = 9.8 N kg^{-1}.

Answer

Moments about *A* = (10 000 kg × 9.8 N kg^{-1} × 3) + (2000 kg × 9.8 N kg^{-1} × [6.0 m – 2.0 m]) = 294 000 N m + 78 400 N m

Moments about *A* = 372 400 N m

Moments about *B* = (10 000 × 9.8× 3) + (2000 × 9.8 × 2) = 294 000 N m + 39 200 N m

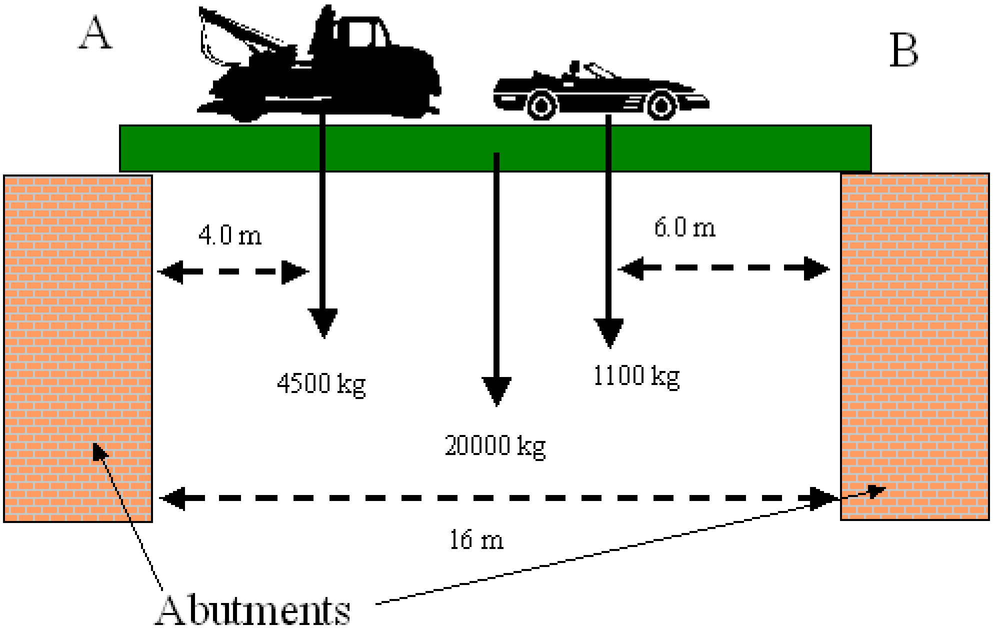
Moments about *B* = 333 200 N m

Force on *A* = 333 200 N m ÷ 6.0 m = **55 533 N = 53 000 N (2 s.f.)**

Force on *B* = 372 400 N m ÷ 6.0 m = **62 067 N = 62 000 N (2 s.f.)**

Now let's do an example with two vehicles on. The principles are the same.

Worked Example



On the bridge shown in the diagram above, what are the forces on the abutments *A* and *B*?

Use g = 9.8 N kg^{-1}.

Answer

To work out the force on *A*, we need to take moments about *B*.

The lorry is 4.0 m from *A*, while the car is 16 m - 6.0 m = 10 m from *A*

The lorry is 16 m - 4.0 m = 12 m from *B*, while the car is 6.0 m from *B*

Moment made by the centre of mass of the bridge = 20000 kg × 9.8 N kg^{-1} × 8.0 m = 1 568 000 N m

Moment made by the car = 1100 kg × 9.8 N kg^{-1} × 6.0 m = 64680 N m

Moment made by the lorry = 4500 kg × 9.8 N kg^{-1} × 12 m = 529 200 N m

Total moments = 1 568 000 N m + 64680 N m + 529 200 N m = 2 161 880 N m

Force on *A* = 2 161 880 N m N m ÷ 16 m = 135 117 N **= 140 000 N** (2 s.f. as the data are to 2 s.f.)

To work out the force on *B*, we need to take moments about *A*:

- Moment made by the centre of mass of the bridge = 20000 kg × 9.8 N kg^{-1} × 8.0 m = 1 568 000 N m
- Moment made by the car = 1100 kg × 9.8 N kg^{-1} × 10 m = 107 800 N m
- Moment made by the lorry = 4500 kg × 9.8 N kg^{-1} × 4.0 m = 176 400 N m
- Total moments = 1 568 000 N m + 107 800 N m + 176 400 N m = 1 852 200 N m
- Force on *B* = 1 852 200 N m ÷ 16 m = 115 763 N = **120 000 N** (2 s.f.)

Note that we have given our answer to 2 significant figures, as the data are to 2 s.f.

Many other problems involving moments can be solved easily by modelling them as bridge problems, for example:

- A table with objects on it.
- A motorbike (the centre of mass is not in the middle in this one).
- A lorry carrying a load.
- A bridge crane.

Tutorial 5.05 Questions

5.05.1

A bridge is 8.00 metres long and has a mass of 20 000 kg. Calculate:

a. The weight of the bridge.
b. The moment about each end of the bridge.
c. The force acting on each end of the bridge.

State what assumption you made.

Use g = 9.81 N kg^{-1}.

5.05.2

In the bridge below (*Figure 67*) which is 6.00 m in length, a car of mass 1500 kg is 1.50 m from side A, and a van of mass 2500 kg is 1.75 m from side B. What forces are acting on sides A and B?

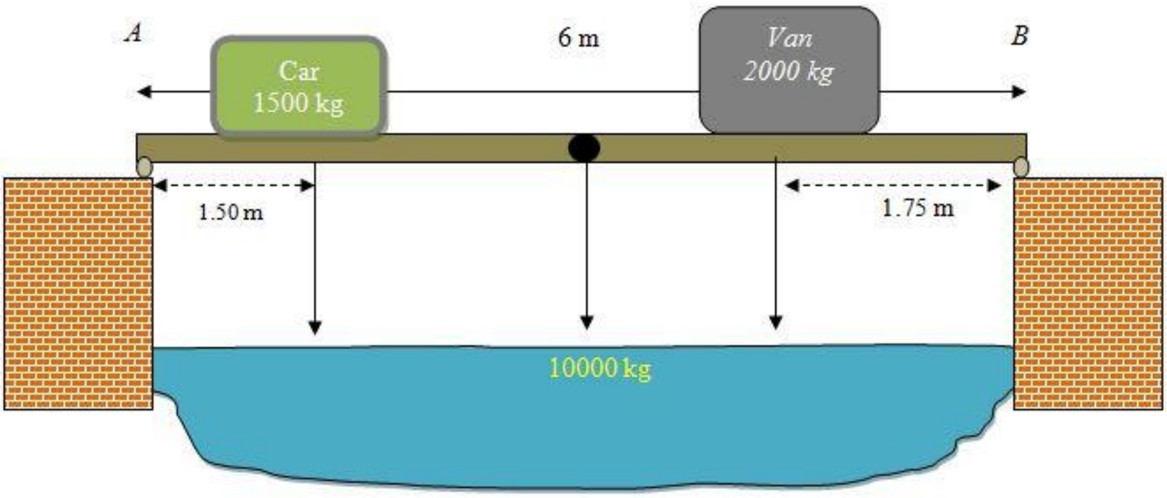


Figure 67 Diagram for 5.05.2

Give your answer to an appropriate number of significant figures.

Use g = 9.8 N kg^{-1}.

Part 2 Motion

Tutorial 5.06 Motion in a Straight Line

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5.061 Speed, Velocity, and Acceleration

This topic looks at **linear motion**, i.e. motion in a straight line.

- **Distance** is how far you travel between any two points by any route. It is a **scalar** quantity.
- **Displacement** is the minimum "as the crow flies" distance between two points. It is a **vector** quantity, so it has **direction**.
- **Speed** is how fast you go, **the rate of change of distance**.
- **Velocity** is **rate of change of displacement**. It must have a direction.
- **Acceleration** can be used as both a vector and a scalar quantity. It is the **rate of change of speed or velocity**.

You will be familiar with the simple equations:

$$\text{speed (m s}^{-1}\text{)} = \frac{\text{distance (m)}}{\text{time (s)}}$$

$$\text{velocity (m s}^{-1}\text{)} = \frac{\text{displacement (m)}}{\text{time (s)}}$$

In Physics code this is written:

$$v = \frac{\Delta s}{\Delta t}$$

.... *Equation 29*

We also know acceleration as:

acceleration (m s^{-2}) = change in velocity (m s^{-1}) ÷ time (s)

In Physics code:

$$a = \frac{\Delta v}{\Delta t}$$

.... *Equation 30*

In both these equations, the strange triangular symbol, Δ, is Delta, a Greek capital letter 'D', which is the physics code for "change in". The picture (*Figure 68*) below shows the difference between **distance** and **displacement**.

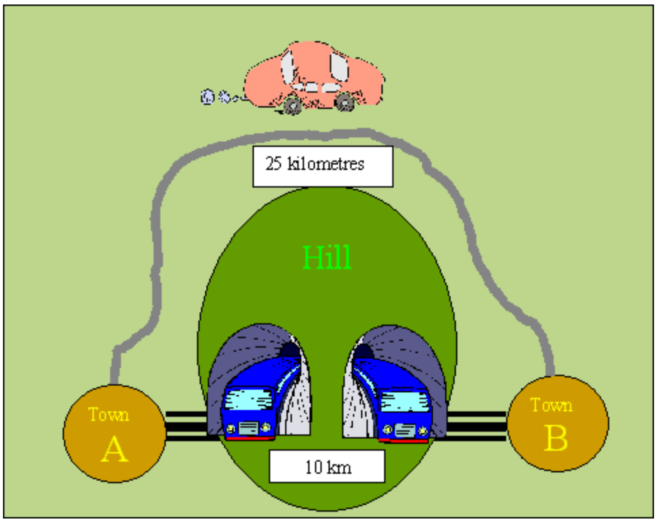


Figure 68 Distance and displacement

Suppose we have two towns A and B 10 km apart but either side of a hill. They are joined by a railway line that is straight and goes through the hill in a tunnel. The road goes round the hill, and the total journey distance is 25 km.

So, the distance is 25 km. The displacement (the straight-line distance in a particular direction) between A and B is 10 km due East.

If we go from A to B and back again, the distance is 50 km, but the displacement is 0.

Information and Communication Technology (Computers) can be used to demonstrate the motion of a vehicle. **Sensors** (light gates or ultrasonic detectors) are connected to a computer, and the computer will record the data at time intervals. The computer will plot a graph of the motion.

Here are some important **definitions**.

Distance. How far two places are apart, by whatever route

Displacement

The straight line distance between two points in a particular direction

Speed

Speed = distance ÷ time

Velocity

Velocity = displacement ÷ time

Acceleration:

Acceleration = change in velocity ÷ time

Note that some textbooks (and examination boards) use "velocity" as a posh word for speed. It has a specific meaning, **displacement divided by time**.

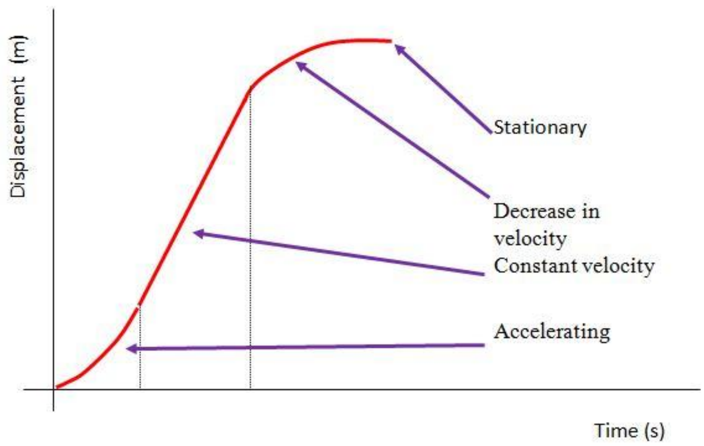
5.062 Graphical Interpretation of Acceleration

We can represent the movement of objects using a **graph**, usually plotting **time** on the *x*-axis (horizontal) and the speed or distance on the *y*-axis (vertical).

- These are a good way of showing the motion of an object with time.
- Time is always on the horizontal axis.
- Motion is always on the vertical axis.
- When looking at a motion time graph, you must be careful about what the motion is. LOOK at the vertical axis.
- In the exam, the graphs that need numerical analysis will be of uniform motion.
- You may be asked to sketch graphs for non-uniform motion.

Displacement (Distance) - Time Graph

Here is a **displacement-time graph** (*Figure 69*)



The gradient of the displacement time graph is the velocity.

If the graph goes up as a straight line the velocity is constant.

The area under the graph has no meaning.

Figure 69 A displacement-time graph

For a distance time graph, the gradient would be the **speed**.

Velocity (Speed) - Time Graph

Consider a train accelerating from a station along a straight and level track to a maximum velocity and slowing down to a stop at the next station. The easiest way to show this is with a **velocity time** graph (*Figure 70*).

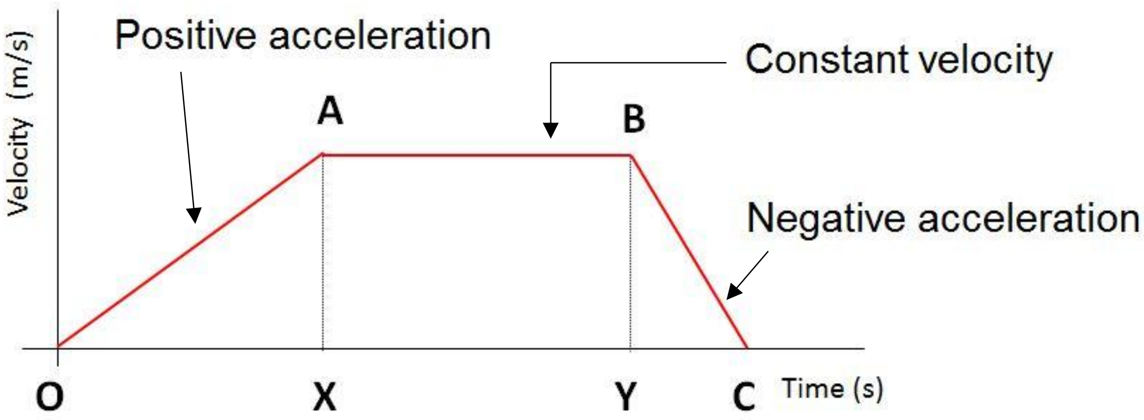


Figure 70 Velocity-time graph

Acceleration is the **gradient** of the speed-time graph.

From the graph,

- between O and A, the train is **accelerating**.
- between A and B, the train travels at a **constant velocity** (speed).
- between B and C, the train **slows down**. Slowing down can also be called **negative acceleration**, or **deceleration**. It is given a **minus sign**.

Displacement (Distance) is the area under the speed-time graph. To work out the total distance, we would add the areas of:

- triangle OAX.
- rectangle ABXY.
- triangle BCY.

Remember to make sure you know whether the problem asks you to consider a **speed-time graph** (as in the case above, or whether it is a **velocity-time graph**. The velocity time graph will take **direction** into account.

The corresponding distance (displacement) time graph is like this (*Figure 71*).

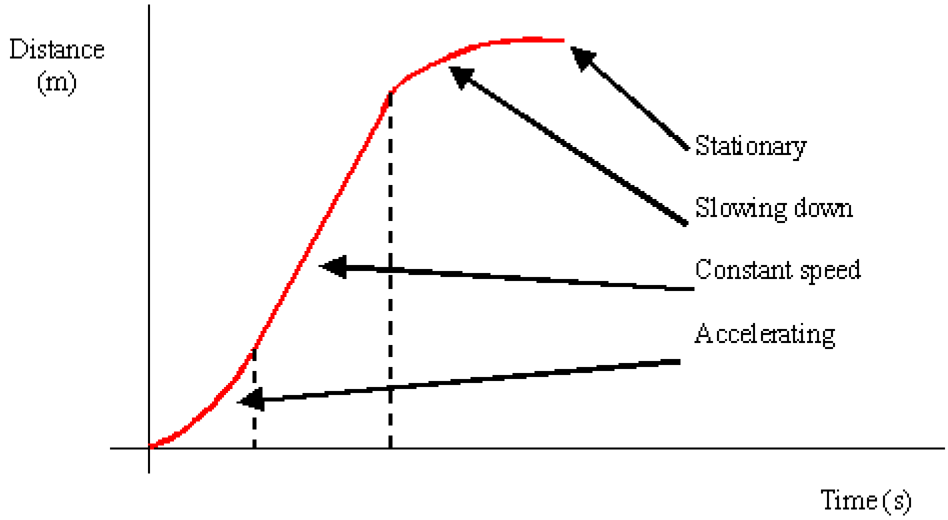


Figure 71 Distance time graph for a train

We can work out the speed at any instant by measuring the **gradient** of the distance time graph. The curved line tells us that the speed is changing.

Acceleration-Time Graph

Acceleration is usually **uniform**, which means that the speed [velocity] is changing at a **constant rate**. However, in many real-life situations acceleration is not constant. Therefore, the speed [velocity] time graph is not a straight line (*Figure 72*).

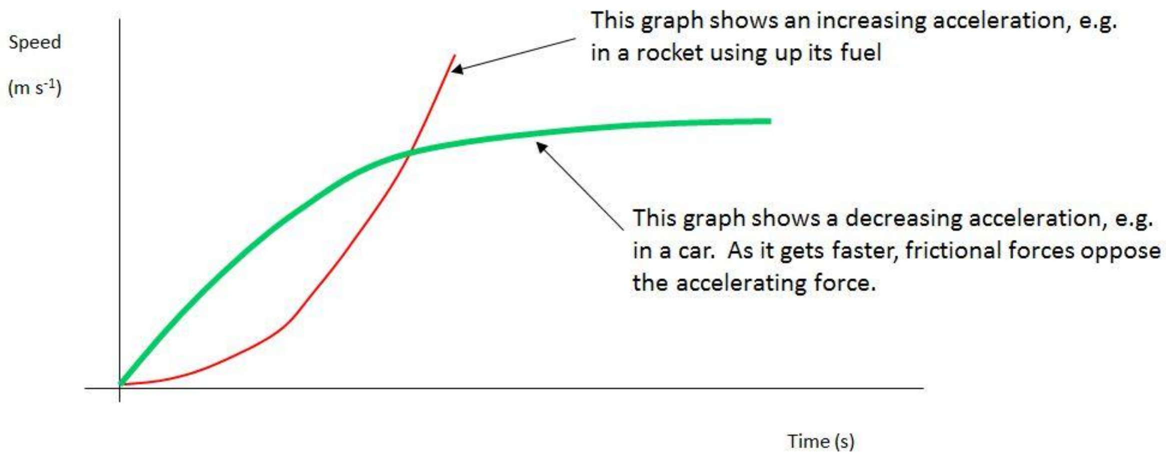


Figure 72 Speed time graph showing non-uniform acceleration.

Here is an acceleration-time graph for a rocket (*Figure 73*).

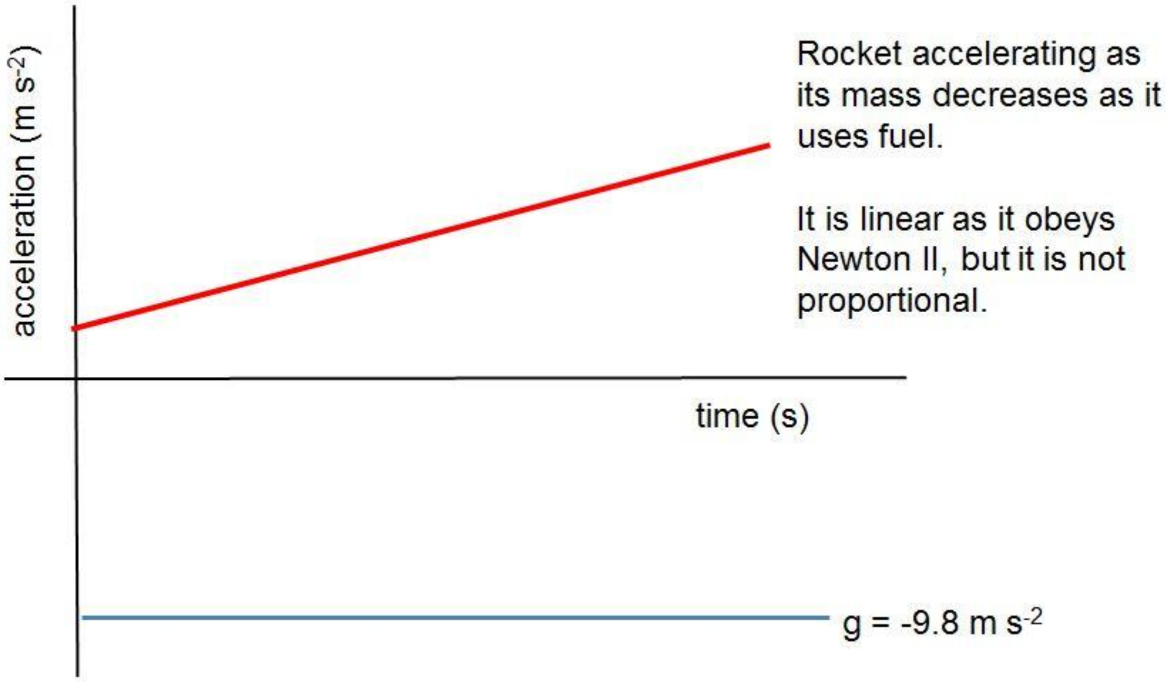


Figure 73 Acceleration-time graph showing uniform and non-uniform acceleration.

Look at the red line. When the rocket is full of fuel at $t = 0$, it will still have a certain value of acceleration. If it doesn't, it will stay on the ground (and probably explode), which rather defeats the object of having a rocket. So, the graph is NOT proportional, although it is linear.

If the acceleration is **constant** (blue line) it will be a **horizontal** straight line. In this case, the blue line represents the **acceleration due to gravity**, g = -9.8 m s^{-2}. The minus sign shows that the direction is **downwards**.

If the acceleration is **positive**, it means that velocity is **increasing**. If the acceleration is **negative**, the velocity is **decreasing**. The term **deceleration** can be used with speed, but not velocity, for which the correct term is **negative acceleration**.

The **area under the graph** represents the **velocity**. If the area is **positive**, it means **upwards**. If it's **negative**, it means that the velocity is **downwards**.

The **gradient** of the acceleration-time graph has no meaning.

5.063 Instantaneous and Average Velocity

It is important to distinguish these two quantities. Instantaneous velocity is worked out using the **gradient** of the **tangent** (*Figure 74*).

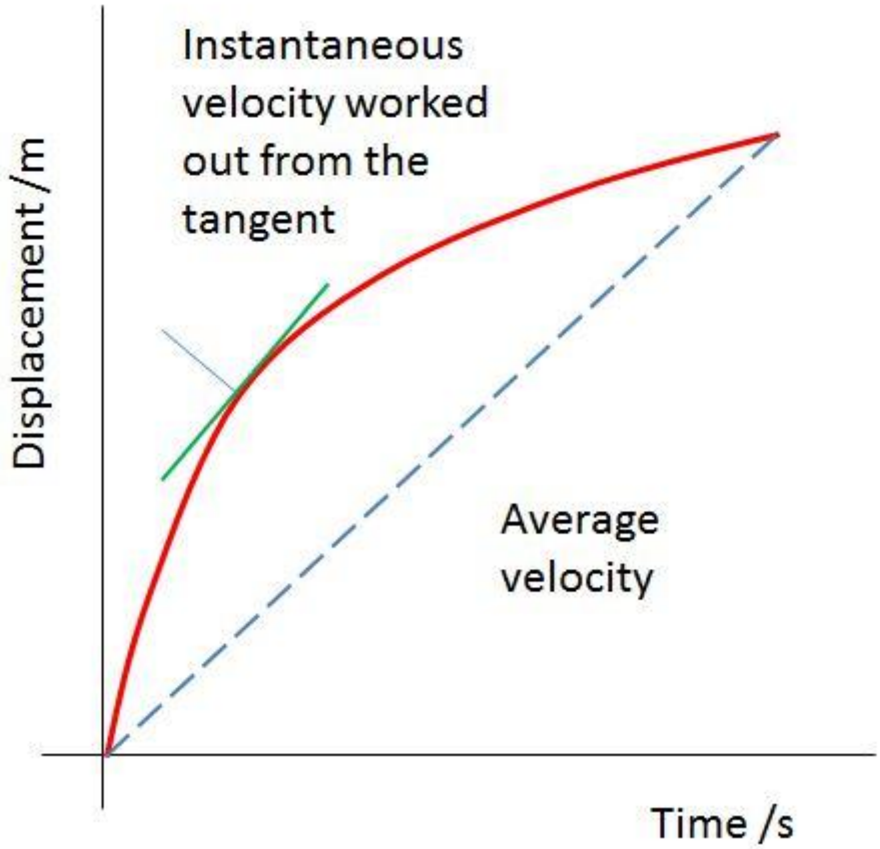


Figure 74 Distinguishing instantaneous and average velocity.

The **instantaneous** velocity gives the velocity at a **particular moment in time**. It will not be the same as the **average** velocity. The **average velocity** is the **total** displacement divided by the **total** time period.

5.064 Emergency Stops in the Car

A good and safe driver should rarely have to brake hard, let alone make an emergency stop. Sometimes the unexpected does happen and you need to know how to stop really quickly and safely. Many of you will practise this in your driving lessons and do it when you go on test. We need to know the way that affects the speed-time graph.

Suppose you are driving at a speed of v m s^{-1}. You see something happen (e.g. a car pulls out in front of you). **Reaction time** for a human is typically about 1/3 s. We will call it t_r. So, the **reaction distance** is given by:

$$s_r = vt_r$$

Then there the time taken for you to decide to stop and apply the brake. It is called **thinking time**. That may be about 1/3 s, about the time taken to shout an explosive expletive. We will call it t_t So the **thinking distance** is given by:

$$s_t = vt_t$$

Once you apply the brake, the car slows to a stop. This takes a particular time, t_s the **stopping time**. This distance is given by:

$$s_s = vt_s/2$$

The **total distance** is the sum of these distances:

Total distance = reaction time + thinking time + stopping time

This is summed up as the **area under the graph** (*Figure 75*).

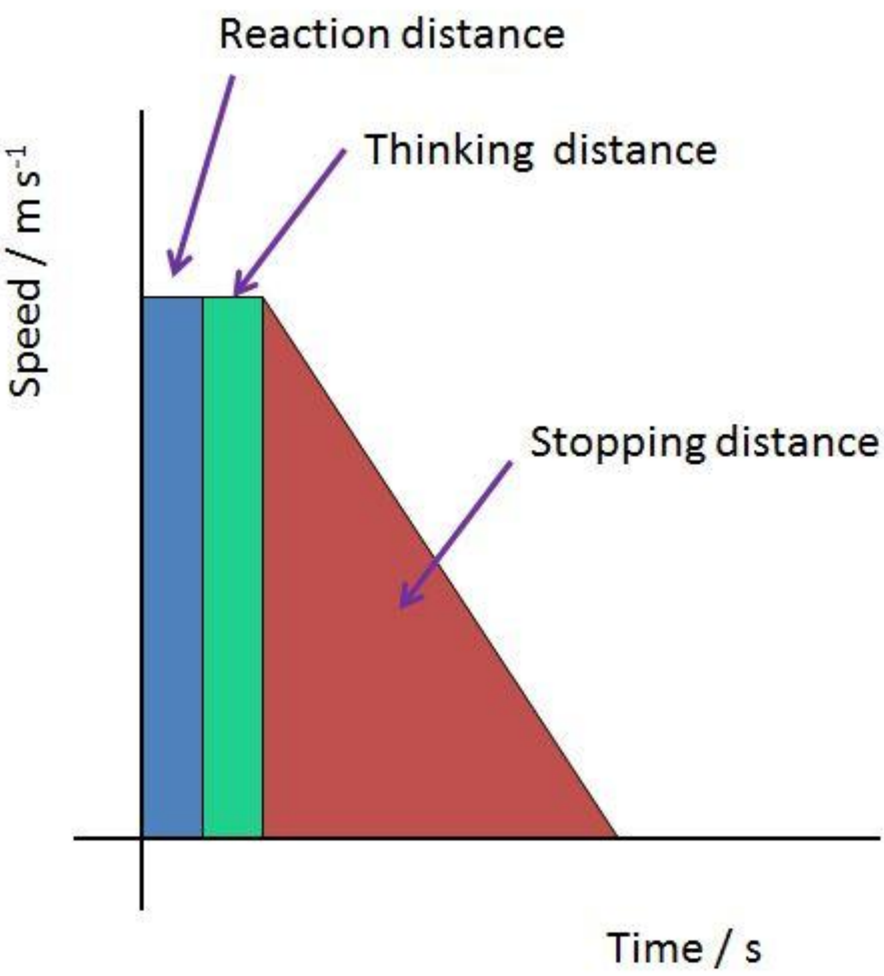


Figure 75 Stopping a car in an emergency.

Some people unthinkingly take to the roads **impaired** by alcohol or drugs. The graph for such a driver is more like this (*Figure 76*).

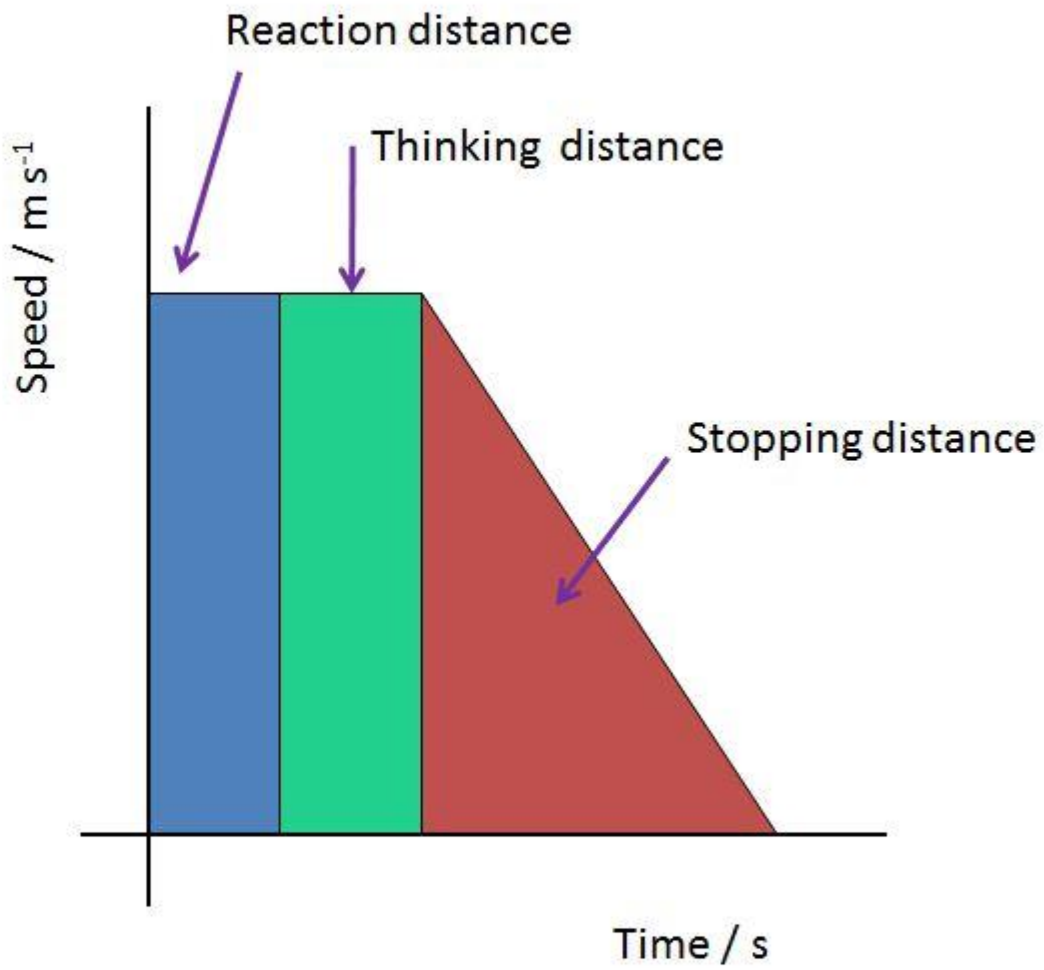


Figure 76 The effect on the stopping distance for an impaired driver.

The stopping distance can be affected by the condition of the car (e.g. the brakes) or the state of the road (is it wet or icy?)

5.065 Equations of Motion

We can use the **Equations of Motion** to calculate the speed of an object under different circumstances. They are sometimes called the **kinematics equations**. These are quantities are involved in **linear motion**, movement in a **straight** line:

Quantity	Physics Code	Units
Distance	s	m
Speed at the start	u	m s^{-1}
Speed at the end	v	m s^{-1}
Acceleration	a	m s^{-2}
Time	t	s

1. Speed at finish = speed at start + change in speed

   change in speed = acceleration × time.

   Speed at end = speed at start + (acceleration × time)

$$v = u + at$$

.... *Equation 31*

2.

$$v^2 = u^2 + 2as$$

.... *Equation 32*

3. Distance = average speed × time

$$s = \frac{(u+v)t}{2}$$

.... *Equation 33*

4.

$$s = ut + \frac{1}{2}at^2$$

.... Equation 34

Maths Window:

Tips for Calculations

1.Write down a formula or equation using standard symbols if possible.

$$s = ut + \frac{1}{2}at^2$$

2.You may wish to write a "shopping list" for the quantities in the formula/equation but this **gets no marks.**

s = 4.5 m; *u* = 0 m; *v* = not mentioned; *a* =?; *t* = 0.90 s.

3.Substitute values into the formula/equation **without rearranging first**

$4.5\ \text{m} = 0\ \text{m} + 1/2\ a \times (0.90\ \text{s})^2$

(unless you know you won't make daft mistakes).

4.Rearrange and calculate your final answer.

a = 4.5 ÷ 0.405

$a = 4.5\ \text{m} \div 0.405\ \text{s}^2$

5.Write the answer using appropriate **significant figures** and, if necessary, standard form. Add the proper **units**.

$a = 11.1\ \text{m s}^{-2}$ = **11 m s**$^{-2}$ (2 s.f. as data are to 2 s.f.)

Check that you have answered what the question actually asked for.

If you have done all this underline your answer.

In many examples we can ignore air resistance, although you will know for yourselves that the faster you go on a mountain bike, the harder you have to pedal. This is because of the effects of **friction** and **air resistance** (drag). We will look at this next in **Terminal Velocity**.

5.066 How are the equations of motion derived graphically?

All the equations of motion are derived from the **speed (or velocity) time graph** (*Figure 77*).

1. The first one is quite easy as it's derived from the area under the graph:

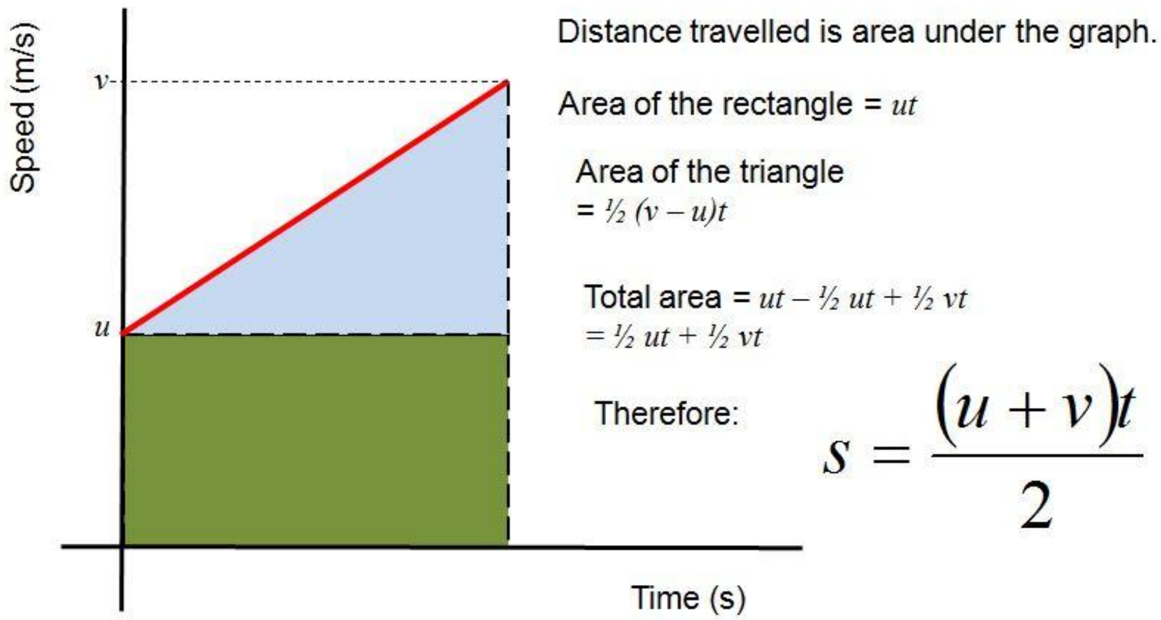


Figure 77 Deriving the distance from the change in speed

2. This graph shows acceleration (*Figure 78*):

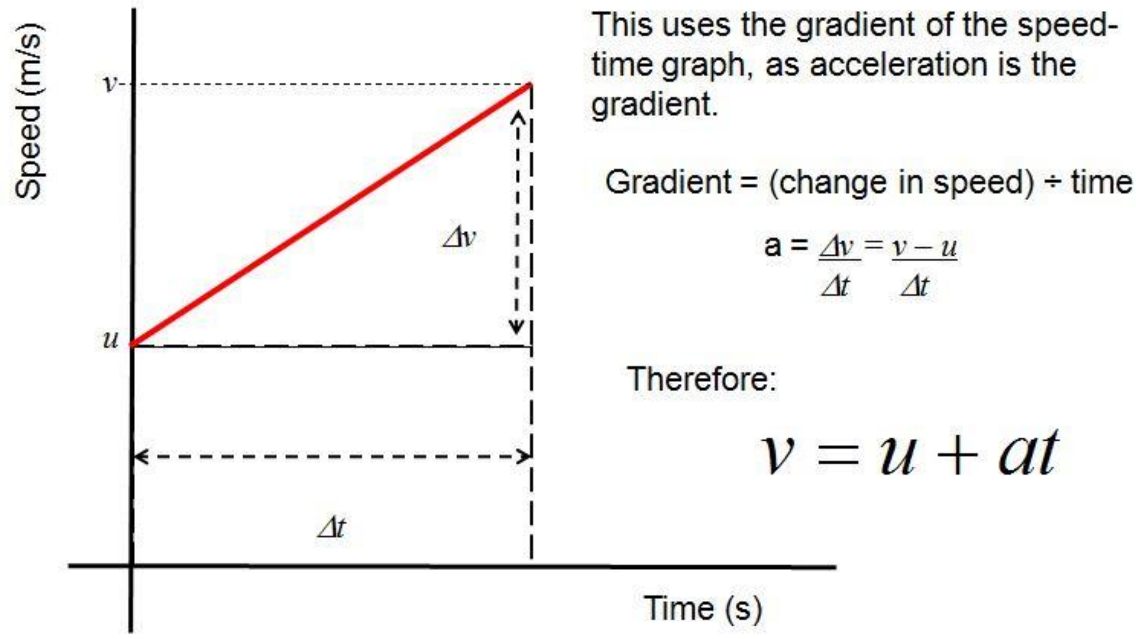


Figure 78 Deriving v = u + at

3. This derivation is a little more complex to understand (*Figure 79*):

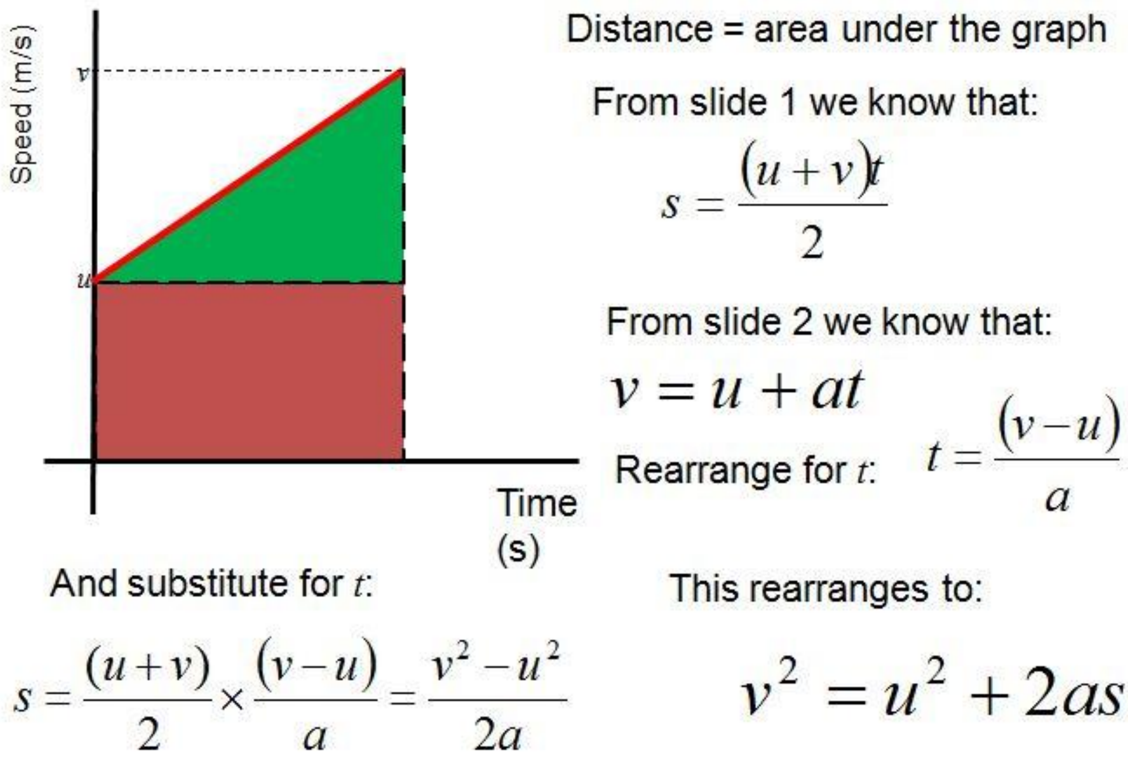


Figure 79 Deriving $v^2 = u^2 + 2as$

4. The final derivation requires some manipulation of equations (*Figure 80*).

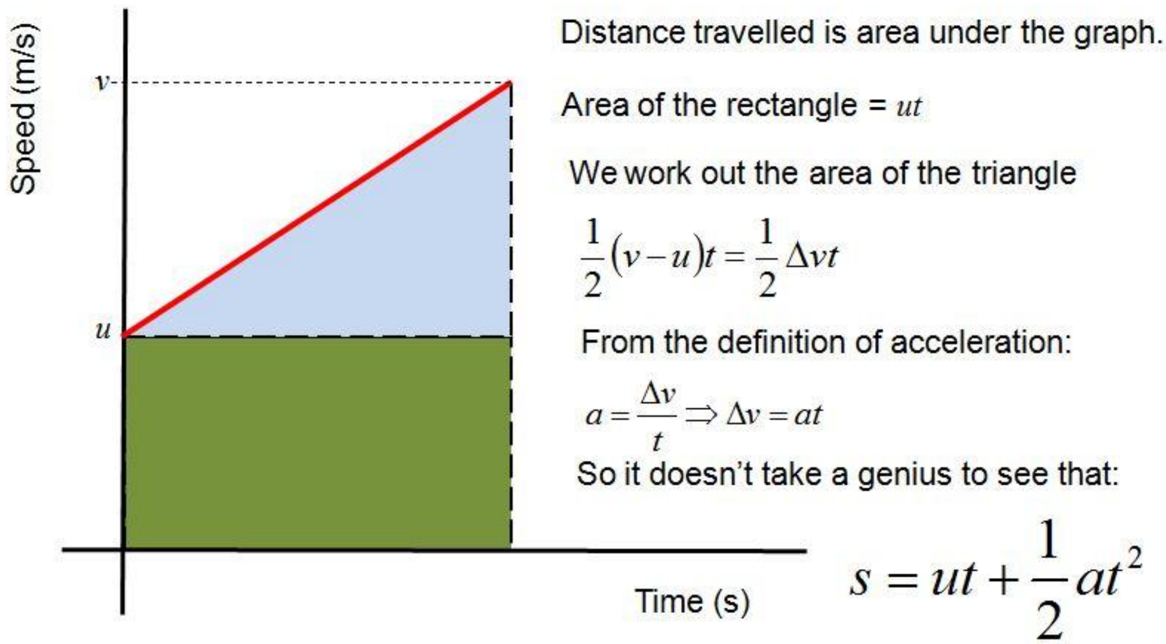


Figure 80 Deriving s = ut + 1/2 at^2.

5.067 How do we select the right equation?

The flow chart below (*Figure* 81) can be used to help you to select the correct equation of motion to use:

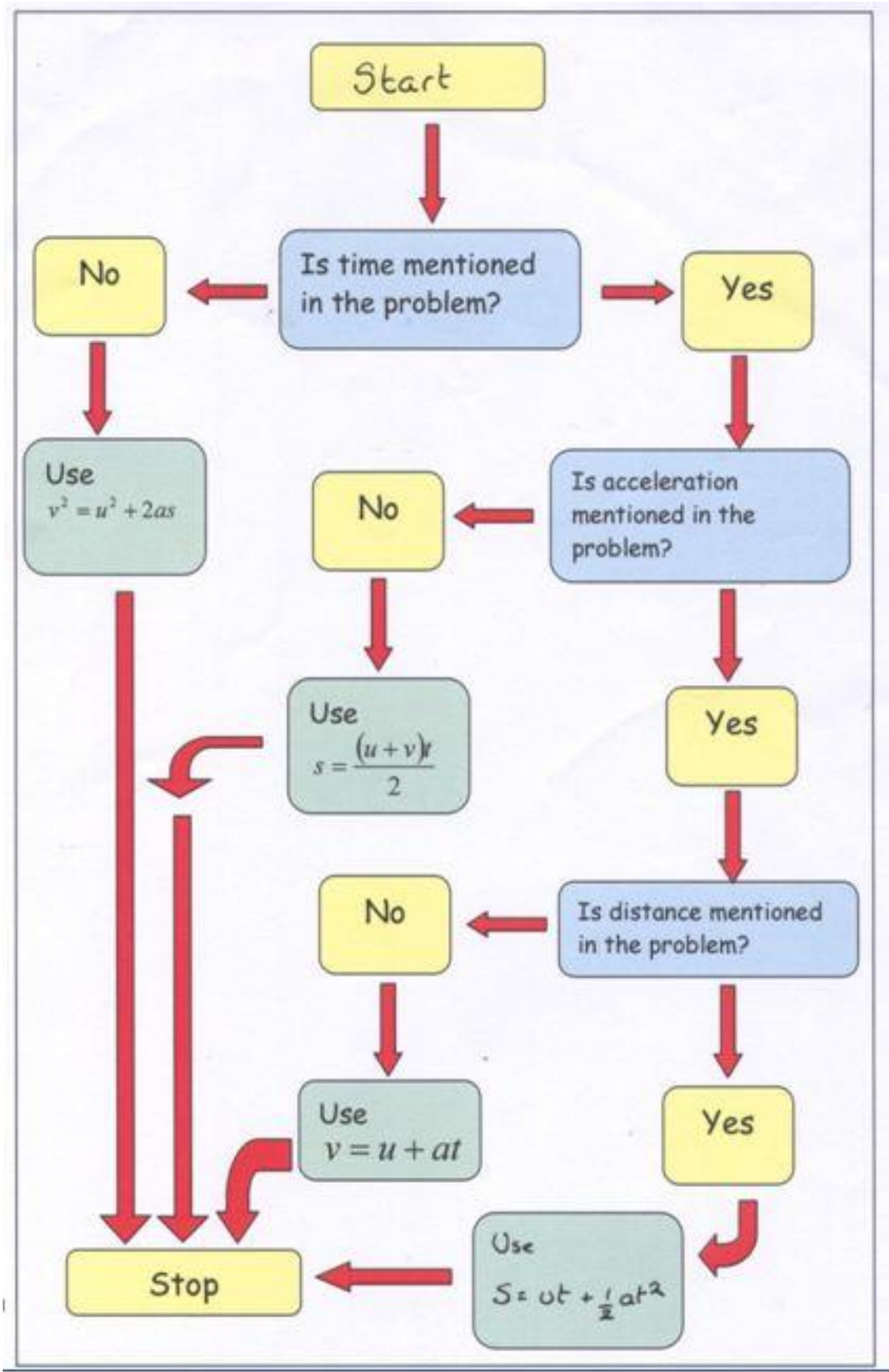


Figure 81 Flow chart to select the right equation of motion.

5.068 Using Calculus in Kinematics (Extension)

In the 1st Year (AS) exam, you are NOT expected to use calculus. In the Scottish Advanced Higher syllabus, you will need to know about this. Most physics A-Level students will do Mathematics, so should be quite familiar with this material. However, do NOT try to be clever in the exam. If you mess up with calculus where a graphical method would do, you may well find yourself with a physics error, 0 marks!

So far, we have used a **graphical** argument that shows the relationship between displacement, velocity, and acceleration. In simple graphical treatments of the kind we used above, acceleration is counted as **constant**. In reality it is not. We do not get sudden **transitions** from acceleration to constant velocity, with sudden graphical **inflections**. At university level, a **calculus** treatment is preferred. Calculus is a mathematical technique that allows us to work out the gradient of a graph or the area under the graph, if we know the relationship between two quantities.

In kinematics you will see the following:

1. Velocity is the rate of change of displacement. In calculus notation, this is written as:

$$v = \frac{ds}{dt}$$

.... *Equation 35*

This has the general pattern of the formula for velocity.

$$v = \frac{\Delta s}{\Delta t}$$

.... *Equation 36*

2. Acceleration is the rate of change of velocity. In calculus notation, this is written as:

$$a = \frac{dv}{dt}$$

.... *Equation 37*

Again, this is similar to the formula for acceleration we have seen before:

$$a = \frac{\Delta v}{\Delta t}$$

.... *Equation 38*

So, what is the difference? If we are talking about a constant rate of change, there is no difference. However, if the rate of change is variable, the calculus notation is used for an **instantaneous** change. This is summed up in the graph below (*Figure 82*).

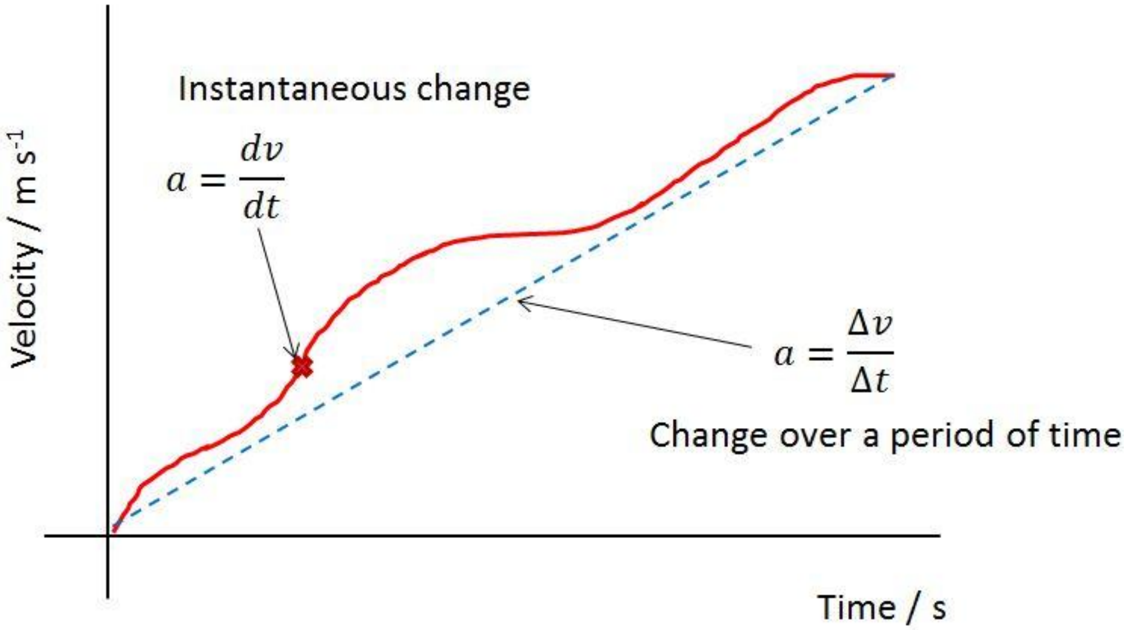


Figure 82 Difference between an instantaneous change and a change over a period of time

In this graph we see a very irregular increase in the velocity of an object. We can look at the rate of change in velocity at a particular instant, or we look at the overall change in velocity over a longer period. The instantaneous change in the velocity is represented by the term dv, while the overall change is represented by Δv.

We could, of course, take a tangent from the graph at the particular instant and measure the gradient of the tangent. However it is likely that there will be uncertainty. With differentiation, there is no uncertainty.

Acceleration is related to displacement using:

$$a = \frac{d_2 s}{dt^2}$$

.... *Equation 39*

This is a **second derivative**, which means a derivative of a derivative.

Using Differentiation

Differentiation is about finding the **gradient** of the graph. As we have seen above:

- **Velocity** is the **gradient** of the displacement time graph.
- **Acceleration** is the **gradient** of the velocity time graph.

Maths Window

There are two things that we can do with Calculus:

1. We can **differentiate**, which means that we find the gradient of the graph of a known relationship.

2. We can **integrate**, which means that we find the area under the graph of a known relationship.

We do these mathematically without having to draw the graph.

This is NOT a comprehensive treatment of calculus, but I hope it will help you how to use it in kinematics calculations

Differentiation

Differentiation is about determining the gradient of a graph.

There are a number of rules of differentiation. We will use only two here.

- Added constants differentiate to 0.
- Powers differentiate according to this formula:

$$\frac{d}{dx}(x^n) = n(x^{n-1})$$

- Multiplied constants are multiplied with the result of the formula that has been differentiated. Suppose the constant is b:

$$\frac{d}{dx}(bx^n) = bn(x^{n-1})$$

Let us suppose we have a straight-line graph that follows the general relationship:

$$y = mx + c$$

If we want to differentiate this, we get:

$$\frac{dy}{dx}(mx + c) = (m \times 1 \times x^0) + 0$$

Therefore:

$$\frac{dy}{dx}(mx + c) = m$$

This tells us that the gradient is m.

We can use calculus to work out the velocity at a particular instant. Here is a graph showing the displacement of an object subject to constant acceleration. We are starting from rest, therefore:

$$ut = 0$$

This graph (*Figure 83*) has been drawn using the equation:

$$s = \frac{1}{2}at^2$$

.... *Equation 40*

The value of the acceleration is 4.0 m s^{-2}.

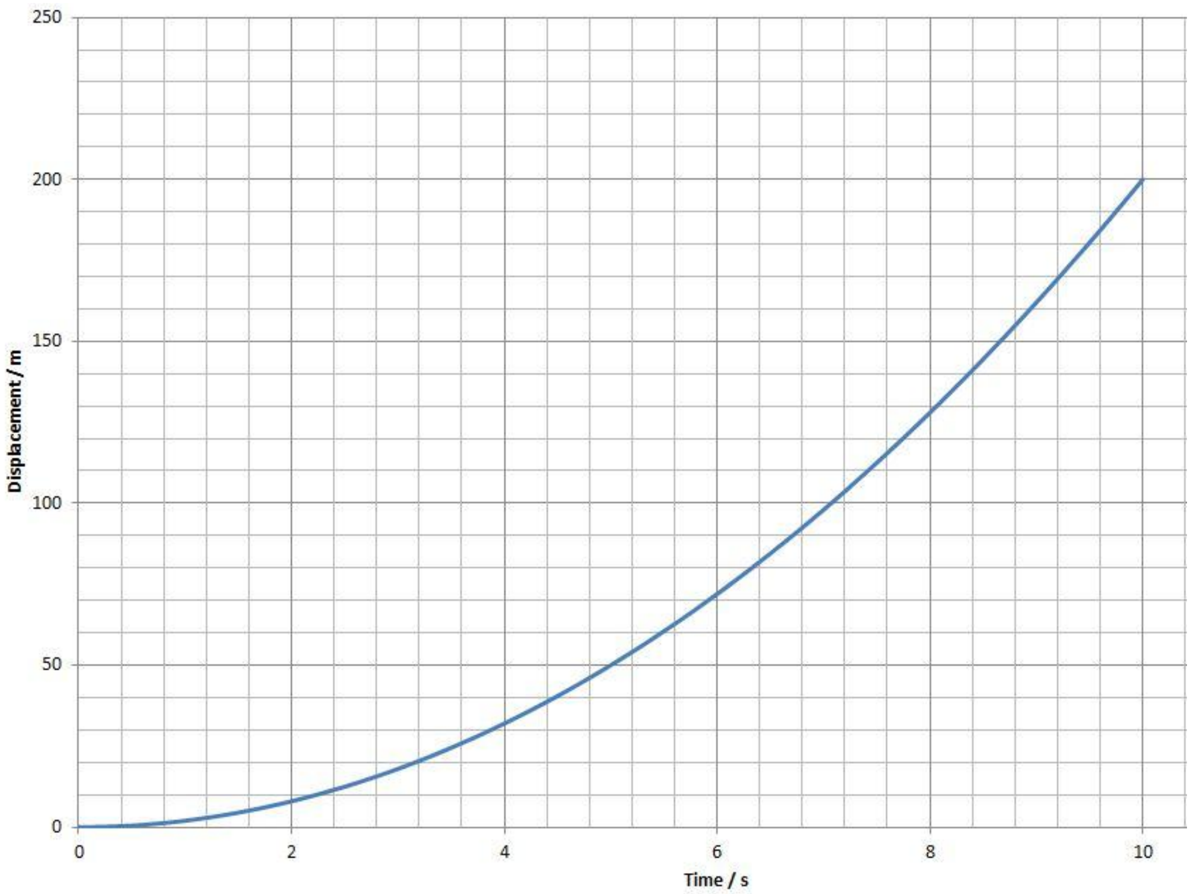


Figure 83 Displacement-time graph with constant acceleration.

We could, of course, work the gradient of the tangent to give us the velocity at exactly 6.0 s, but there will be uncertainty. Instead, let's use calculus:

$$v = \frac{ds}{dt} = 2 \times \frac{1}{2}at = at$$

.... Equation 41

Therefore, we can substitute:

v = 4.0 m s^{-2} × 6.0 s = **<u>24 m s^{-1}</u>**

This is a lot easier that drawing a tangent, then measuring the rise and the run.

We can differentiate the equation:

$$s = ut + \frac{1}{2}at^2$$

.... *Equation 42*

As below:

$$v = \frac{ds}{dt} = 1 \times u \times t^0 + 2 \times \frac{1}{2}at = u + at$$

.... *Equation 43*

To give us the familiar:

$$v = u + at$$

.... *Equation 44*

Using Integration

Integration is about finding the **area under the graph**. We have seen how the kinematics equations have been worked out using the area under the velocity time graph. For some questions we have simply counted the squares under the graph. The counting of square is both tedious and prone to uncertainty. In simple graphical treatments, acceleration is counted as constant. In reality it is not. Therefore, a mathematical approach is more satisfactory, as it can be quicker and is less prone to uncertainty.

Consider a car accelerating at a rate of a m s^{-2} over a period of t s. This is a real-world situation, and the acceleration is not constant, but reduces because of the increase in friction from the road and the air resistance (*Figure 84*).

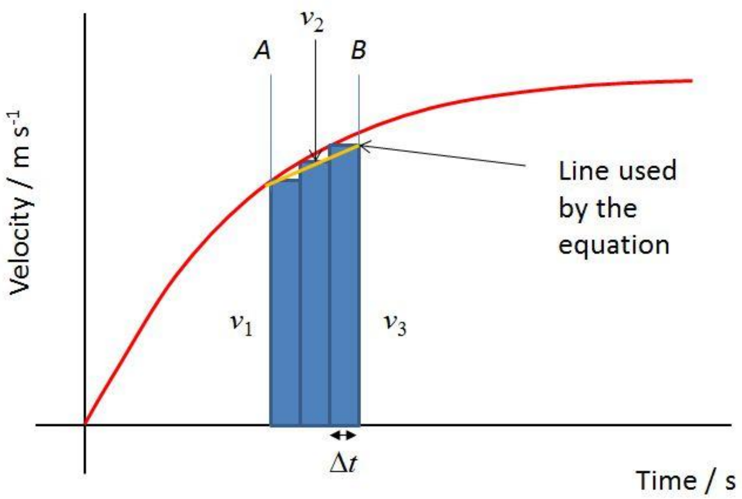


Figure 84 Velocity-time graph of a cat accelerating at a.

Suppose we want to find the displacement *s* between the points *A* and *B*. We know that displacement is the area under the graph. We could use the equation:

$$s = \frac{(v_1 + v_3) \times 3\Delta t}{2}$$

.... *Equation 45*

The equation works out the area under the orange line in the graph. The answer it would give would be too low.

Alternatively, we could break the area into three little strips as shown in the graph. So, we could work out the area of each little strip and add them together:

$$s = v_1\Delta t + v_2\Delta t + v_3\Delta t$$

.... *Equation 46*

This will give us a better answer, but it will only be an approximation. We could make the strips narrower, using a shorter time interval (*Figure 85*).

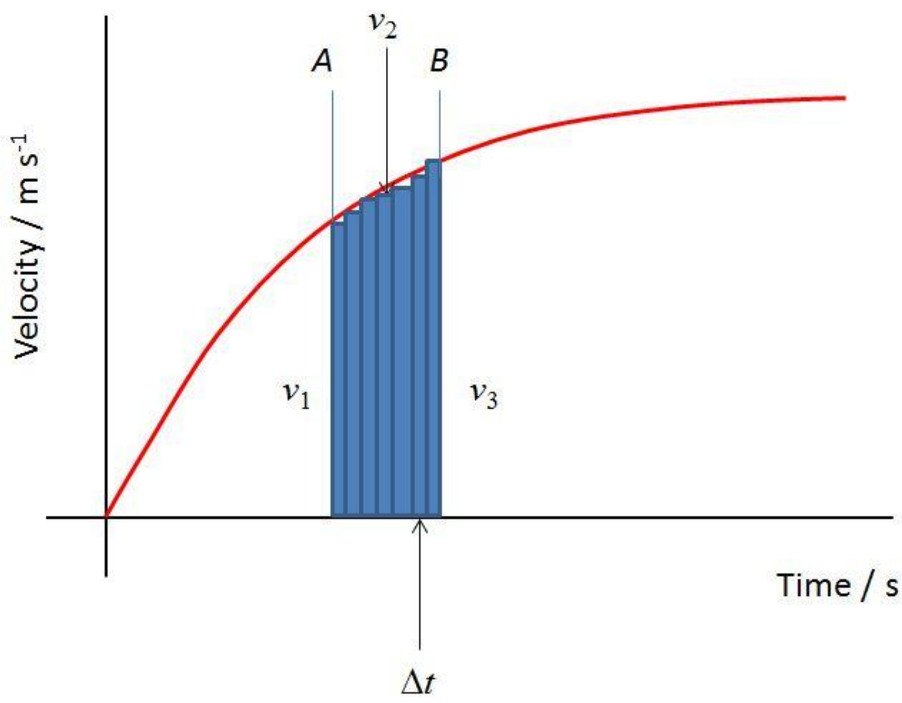


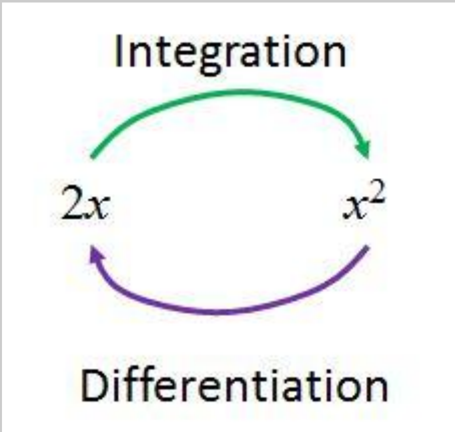
Figure 85 Reducing the width of the little strips.

Therefore, the answer gets closer to the real answer, but it is still an approximation. However, if we make the time interval infinitesimally small, we end up with the true answer. This we do by the process of **integration**. Instead of writing the width of each strip a Δt, we write δt. Integration adds up all the little strips to give us the true answer. The symbol 'δ' is delta, Greek lower-case letter 'd'.

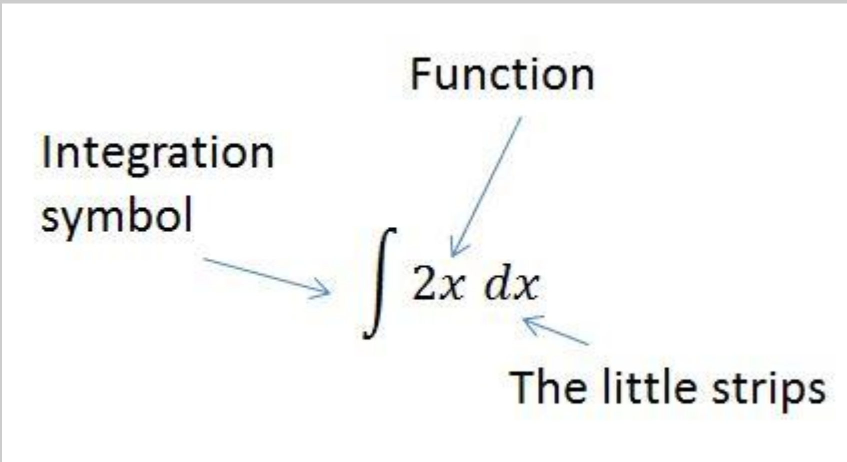
Maths Window

Integration

Integration is the reverse process of differentiation. The idea is shown in the picture below:



The $2x$ term is the **function**. The function to be integrated is sometimes called the **integrand**. Therefore, we write this in calculus form as:



The dx term shows that the little strips go along the *x*-axis. The integration symbol is a fancy capital letter 'S', which means "summed together". So, we now write:

$$\int 2x\,dx = x^2 + C$$

The C term is a **constant**. When we differentiated, the constant that was added to the function had a differentiated value of 0. Now we are applying the process in reverse, we need to have a definite value for the constant.

Here are some rules for integration:

- A constant is added to the integrated function. In some cases, this might be zero. In other cases, it has a definite value.
- Constants that multiply a function are multiplied with the result of the integrated function. Suppose we have a constant, b:

$$\int 2bx\,dx = bx^2 + C$$

- The power rule is *shown* below. It does not work with x^{-1}.

$$\int x^n dx = \frac{x^{n+1}}{n+1} + C$$

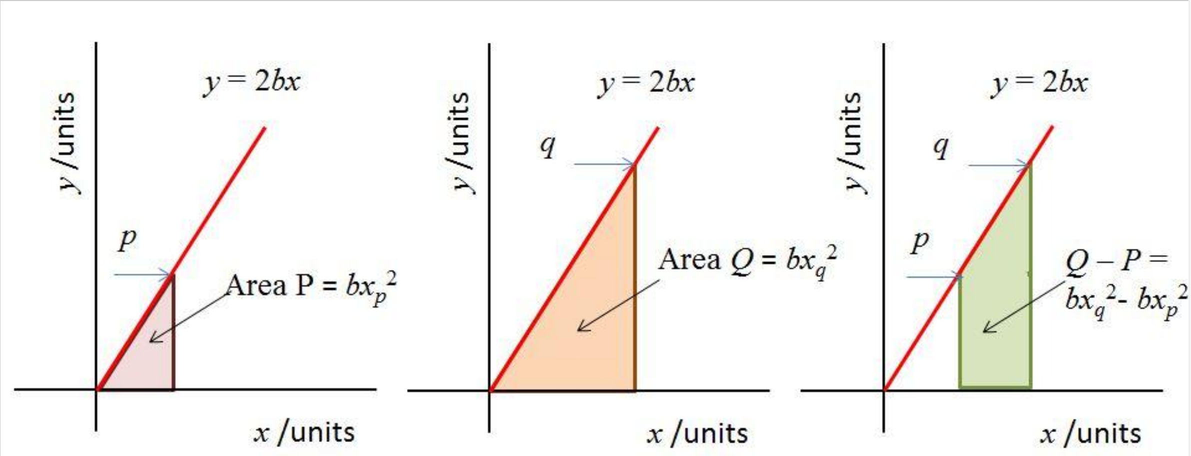
- The integral of x^{-1} is shown below:

$$\int x^{-1} dx = \ln(x) + C$$

Often you will need to integrate between two points. You may see an equation like this:

$$\int_p^q 2bx\,dx = bx^2 + C$$

This means you have to work out the value of the integral at p and the value of the integral at q and then subtract one from the other. This is shown in the picture below. The constant, $C = 0$ for the sake of this argument.



Consider an object moving with a constant acceleration, a m s^{-2} from an initial velocity at $t = 0$, u m s^{-1}, to a final velocity, v m s^{-1} at time t. This is shown on the graph below (*Figure 86*).

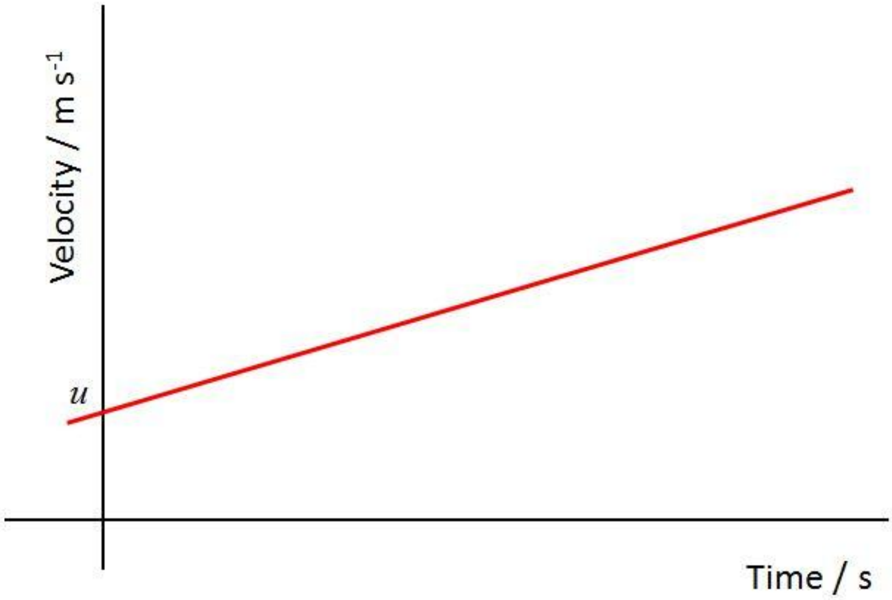


Figure 86 A velocity-time graph with acceleration a

We know that this graph shows the equation:

$$v = u + at$$ .... *Equation 47*

We also know that the area under the graph is the displacement. So, we can integrate, since we know that:

$$v = \frac{ds}{dt}$$

.... *Equation 48*

So, we can write:

$$s = \int_0^\infty u + at\ dt = \frac{ut^{0+1}}{0+1} + \frac{at^{1+1}}{1+1} = ut + \frac{at^2}{2}$$

.... *Equation 49*

Therefore:

$$s = ut + \frac{1}{2}at^2$$

.... *Equation 50*

So, let's use that result:

<u>*Worked Example*</u>

An object has an initial velocity of 6.5 m s^{-1}. At time t = 0 it accelerates at a rate of change of velocity of 2.85 m s^{-1}. What is the displacement between time t = 3.6 s and time t = 8.5 s?

<u>*Answer*</u>

We can write this in calculus notation:

$$s = \int_{3.6\ \mathrm{s}}^{8.5\ \mathrm{s}} 6.5\ \mathrm{m\,s^{-1}} + 2.85\ \mathrm{m\,s^{-2}}t\ dt$$

Therefore:

$$s = \left[(6.5\ \mathrm{m\,s^{-1}} \times 8.5\ \mathrm{s}) + \left(\tfrac{1}{2} \times 2.85\ \mathrm{m\,s^{-2}} \times (8.5\ \mathrm{s})^2\right)\right] - \left[(6.5\ \mathrm{m\,s^{-1}} \times 3.6\ \mathrm{s}) + \left(\tfrac{1}{2} \times 2.85\ \mathrm{m\,s^{-2}} \times (3.6\ \mathrm{s})^2\right)\right]$$

$$s = 158.2\ \mathrm{m} - 41.87\ \mathrm{m} = 116\ \mathrm{m}$$

Tutorial 5.06 Questions

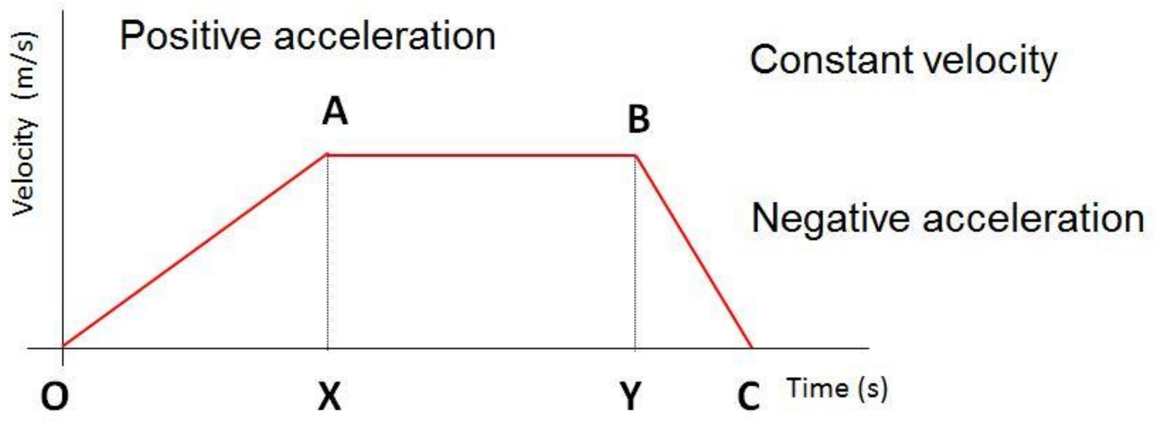
5.06.1

Why is the displacement 0 in the example in pp 72 - 73?

5.06.2

A runner accelerates at a rate of 4 m s^{-2} to her maximum speed of 9.6 m s^{-1}. What is the time taken for her to reach this speed?

5.06.3



Use the information below in the question. The train's motion is that described in the graph above.

- The maximum speed of the train is 25 m s^{-1}
- The time interval OX is 45 s
- The time interval XY is 45 s
- The time interval YC is 20 s

What is:

a) The acceleration between O and A

b) The acceleration between B and C

c) The distance covered while the train is at constant speed

d) The total distance.

e) The average speed?

5.06.4

A car is travelling at 30 m s^{-1}.

It takes 10 seconds to accelerate to a new speed of 35 m s^{-1}. What is its acceleration?

5.06.5

A brick falls off the top of a wall under construction and drops into a bed of sand 14.5 m below. It makes a dent in the sand 185 mm deep. What is:

a) The speed of the brick just before it hits the sand.

b) Its deceleration in the sand.

c) What would happen to a person undergoing that deceleration?

Use g = 9.8 m s^{-2}

5.06.6

An aeroplane of mass 5000 kg lands on a runway at a speed of 60 m s^{-1} and stops 25 s later.

Calculate:

a – the deceleration of the aeroplane.

b – the braking force on the aeroplane.

c - the distance taken by the aeroplane to stop.

Tutorial 5.07 Free Fall and Terminal Speed

All Syllabi

Contents

5.071 Action of Gravity Fields

Gravity is an attractive force between two objects with mass. It is very weak but has an infinite range. The only reason we feel it at all is because the Earth is a very big object with a mass of 5.98×10^{24} kg (which is heavy). Gravity is always attractive. Gravity causes objects with a mass to have a **weight**.

1 kg weighs 9.81 N on the Earth and 1.6 N on the Moon

$$W = mg$$

.... *Equation 51*

The **acceleration due to gravity** on the Earth is **9.81 m s^{-2}**. It is given the physics code g and the value is sometimes approximated to 10 m/s^2. (The latter approximation is perfectly acceptable in GCSE examinations, but you will lose marks if you use it at A-level.) 1 kg weighs 9.81 N on the Earth.

Therefore, we can also write the value of g as **9.81 N kg^{-1}**.

Weight is a force measured in **Newton** (N). It is NOT measured in kilograms. It is depressing how many students make this mistake.

5.072 Motion in a Gravity Field

The acceleration near the Earth's surface remains constant. An object like a ball thrown vertically **upwards** is always accelerating **towards** the Earth at 9.81 m s^{-2}. The split second it leaves the hand, it is accelerating **downwards** at 9.81 m s^{-2}, regardless as to whether the ball is going up or down.

By convention:

- **Upwards** is **positive**.
- **Downwards** is **negative**.

Consider a ball being thrown up **vertically**. The displacement time graph look like this (*Figure 87*). The graph is a **displacement**-time graph, and the direction is important. The path of the ball is straight up and straight down.

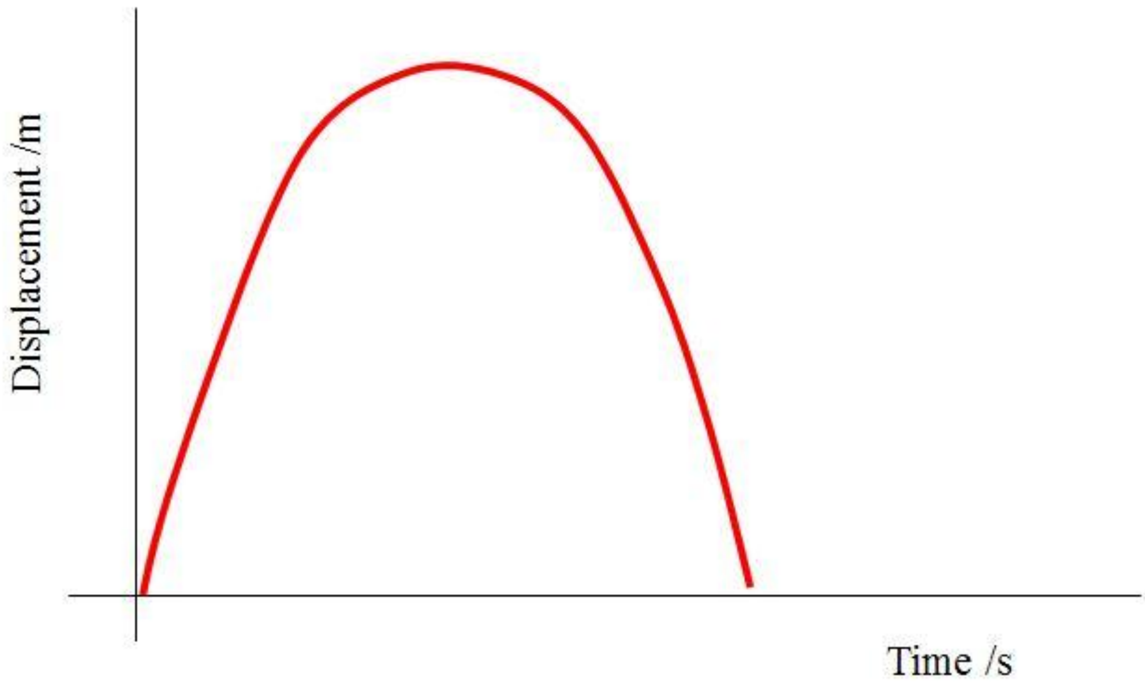


Figure 87 Displacement time graph of a ball being thrown vertically in the air.

We can deduce the following from the graph:

- The ball slows down under a constant **acceleration** of -9.81 m s^{-2} (downwards).
- At the top its velocity is **zero**, but it is still accelerating at -9.81 m s^{-2} (downwards).
- Then it falls downwards at an acceleration of -9.81 m s^{-2}.

We are ignoring air resistance. The graph is a **parabola**. It is very important to take into account of the signs, especially when a ball is being thrown up in the air and then is dropping down again.

The corresponding velocity-time graph is like this (*Figure 88*)

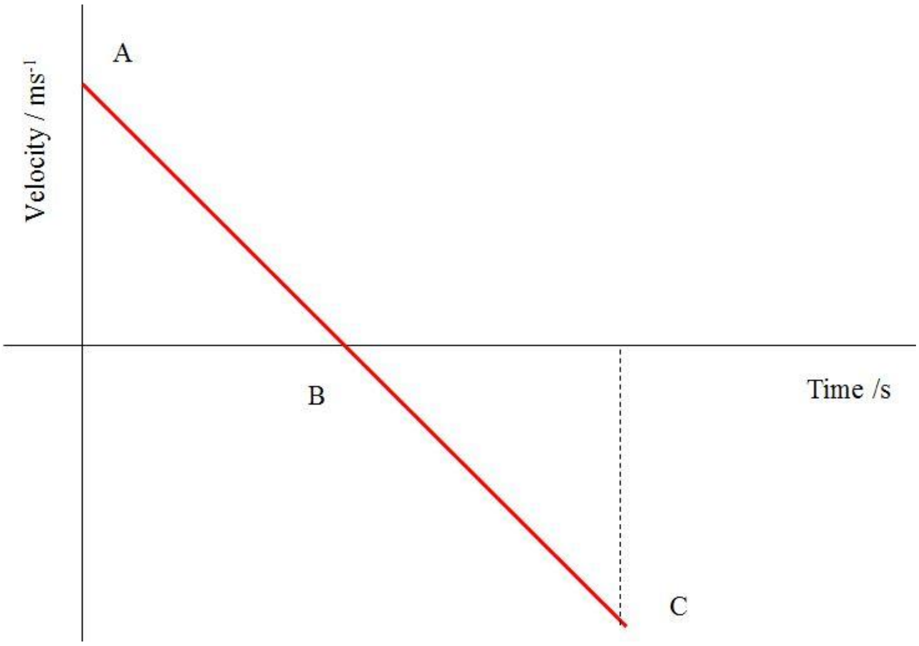


Figure 88 A velocity-time graph for a ball being thrown vertically into the air.

The acceleration is constant and **negative**, so gives a negative gradient. This will have the value of *g*. The velocity will decrease from its highest **positive** value when it leaves the hand. It will fall to **zero** at the highest point and will reach its maximum **negative** value when the ball reaches the ground (or hand).

Mathematically we can write:

$$g = \frac{dv}{dt} = \frac{d_2 s}{dt^2}$$

.... *Equation 52*

This gives rise to the equation:

$$s = ut + \frac{1}{2}gt^2$$

.... *Equation 53*

If the ball is caught at the same height, the **positive** area under the graph is the **same** as the **negative** area, giving a displacement of **zero**. If the ball drops to the ground, the **negative** area is slightly bigger than the **positive** area.

5.073 Measuring g

The **measurement of *g* by free fall** can be done using light-gates and data-loggers, or by timing the drop of a steel ball bearing. This is an **assessed practical** that you will do with your tutor. You will use apparatus like this (*Figure 89*).

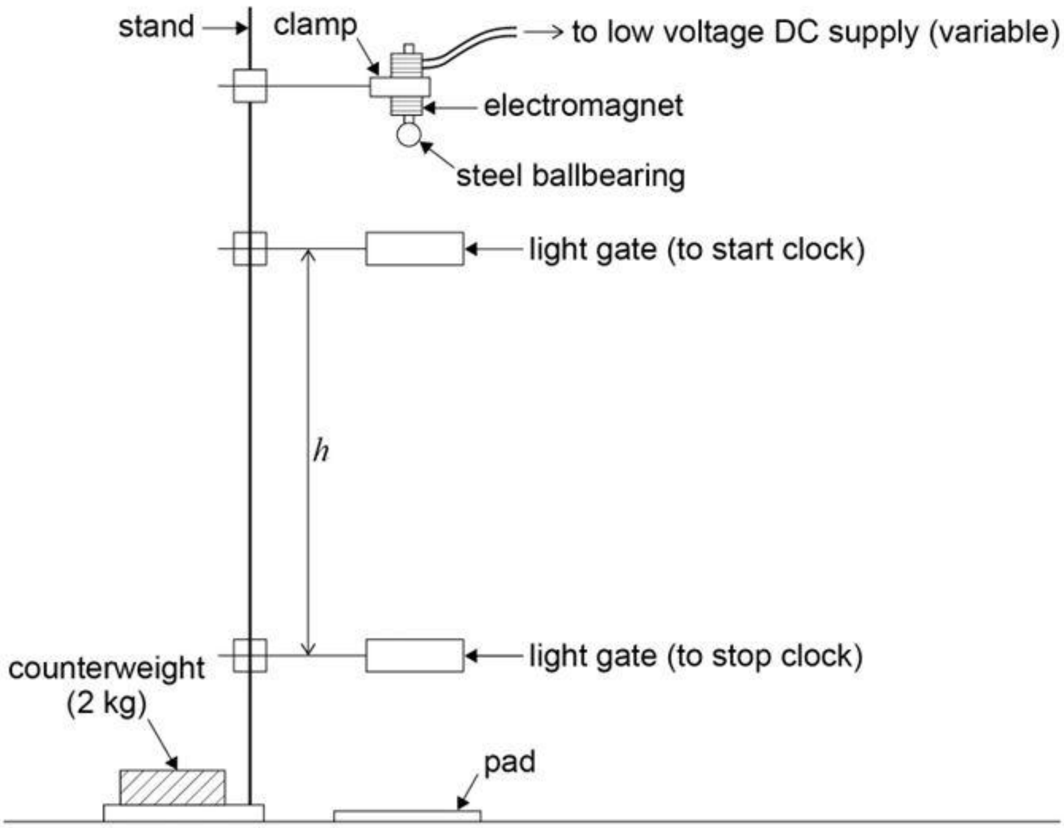


Figure 89 Apparatus to measure g by free fall.

The procedure is this:

- The ball bearing is released when the electromagnet is turned off. It falls through each light gate.
- The data-logger needs to be set to time from A to B.
- You need to vary h, the distance between the light gates.

5.074 Free Fall

All objects will accelerate downwards at (-)9.81 m s^{-2}, regardless of mass. Over a short distance, a small light object will hit the ground at the same time as a larger and heavier object. Over a longer distance, air resistance becomes important. The air resistance reduces the acceleration. The faster the object, the greater the air resistance. Eventually the air resistance balances the weight, and there is zero acceleration. We have **terminal velocity** or terminal speed. Since we are not considering direction, we should call it terminal speed.

For a feather, which has a large surface area compared with its mass, the terminal speed is about 10 cm s^{-1}. For a skydiver, it is about 60 m s^{-1}. For the skydiver when the parachute opens, it is about 5 m s^{-1}. The value of the terminal speed depends on not only the mass of the object, but also its surface area.

The concept of terminal velocity can be applied to an object falling in any **fluid**, i.e. liquid or gas. It does not apply, of course, to solids. Nor does it apply in a **vacuum**. If you throw an object from a spacecraft orbiting the Moon, the object will continue to accelerate at a rate of 1.6 m s^{-2} until it hits the Moon's surface. It might be travelling quite fast.

5.075 Free Fall in Air

Think about skydivers jumping from a plane (*Figure 90*).

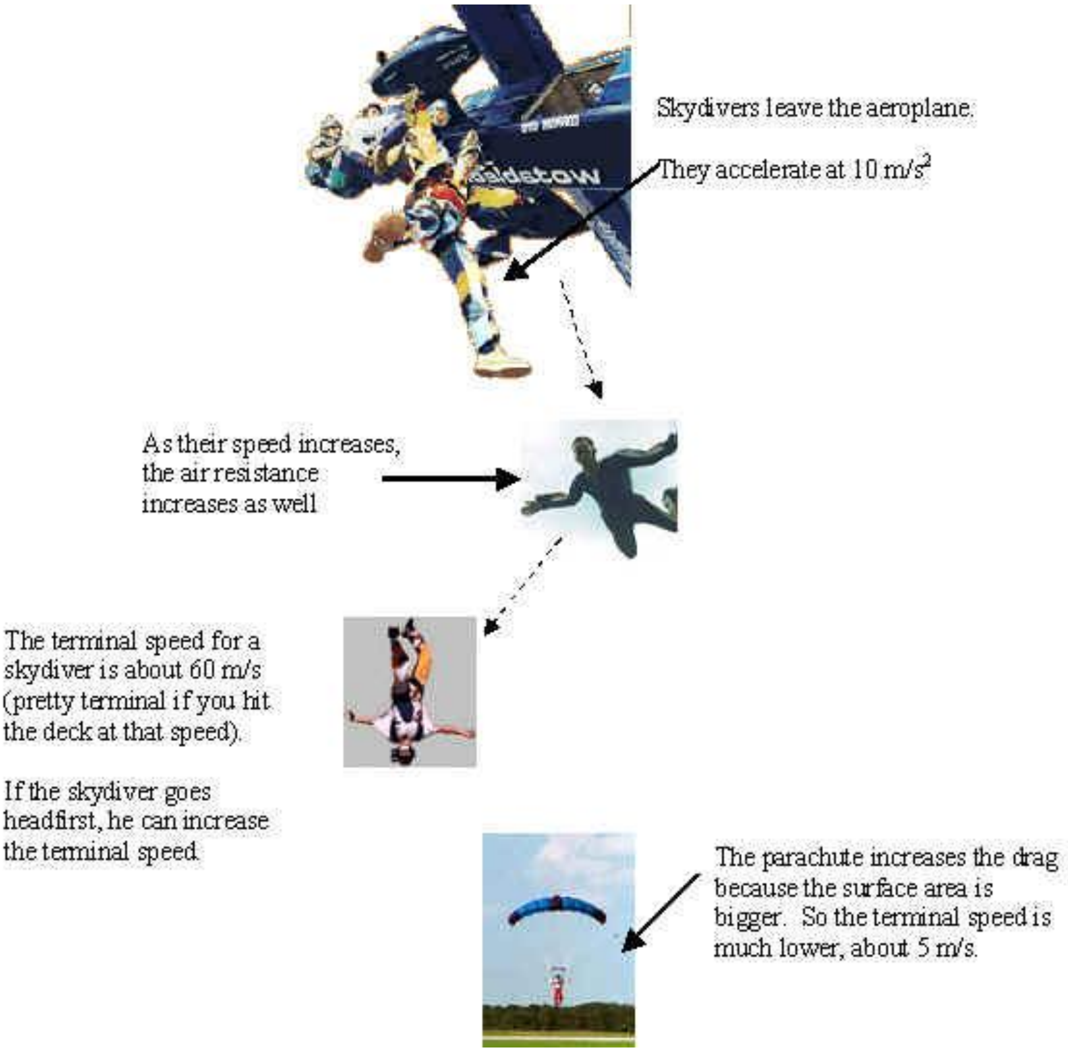


Figure 90 Skydivers in free-fall

The animation will help you understand this (*Figure 91*)

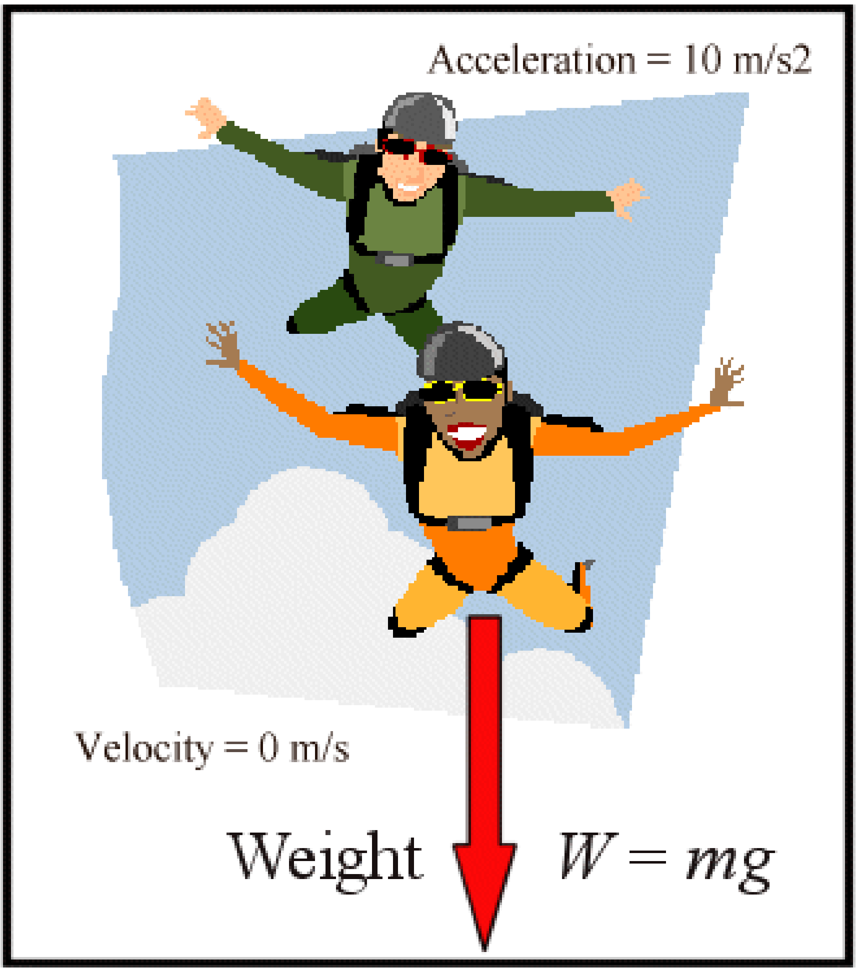


Figure 91 Animation showing force of drag opposing skydivers' weight

The drag is in the opposite direction to the weight, so the acceleration decreases.

Then the drag (upwards force) balances the downward force (weight)

Therefore, there is no acceleration and the speed is constant. This is **terminal speed**.

5.076 Terminal Speed in Liquids (Required Practical)

Dropping objects off tall buildings or lobbing things out of aeroplanes results in certain Health and Safety management issues. It is rather safer to measure terminal speed in a liquid, since the terminal speed is rather lower. And it's easier to time as well. Here is a typical experiment you will do (*Figure 92*).

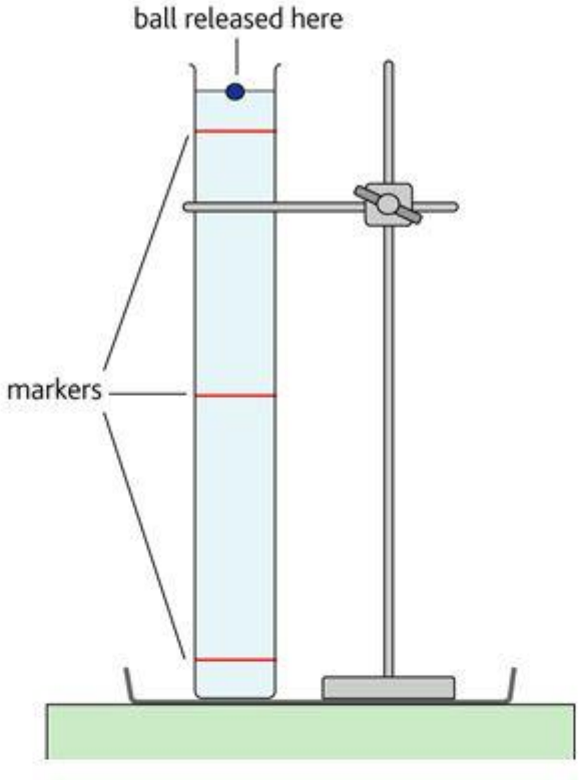


Figure 92 Measuring terminal speed in a viscous liquid.

When a ball bearing is dropped into a **viscous** (gooey) liquid, it almost immediately reaches its terminal speed, and measuring it is simply a matter of timing the motion between two fixed points a known distance apart. You will also have to:

- Note the mass, m, of each ball.
- Use a micrometer to measure the diameter, d.

A **viscous** material is gooey. Runny materials have low **viscosity**. There are various ways of measuring viscosity, the most common of which is to drop a ball-bearing into the liquid and measuring its **terminal speed**. Remember that at terminal speed, the upwards forces of **upthrust** and **drag** and the downwards force of the **weight** are **balanced** (*Figure 93*).

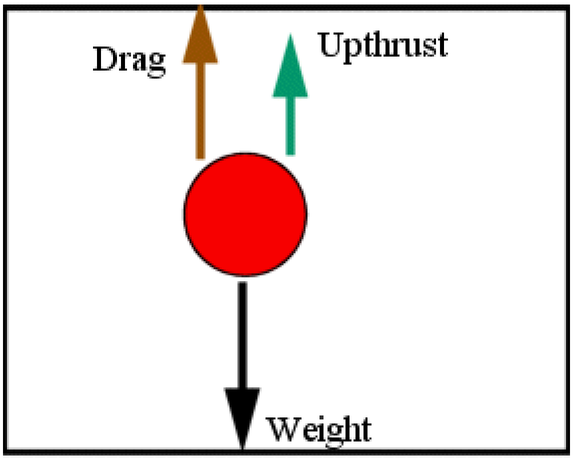


Figure 93 Terminal speed in a viscous fluid.

The upthrust is the same as the weight of fluid displaced by **Archimedes' principle**. Therefore, if the weight is greater than the upthrust, the object will accelerate downwards until the drag balances the difference between the weight and the upthrust. This is true of all fluids, for example air, or water, or chocolate.

We can calculate the drag force by **Stokes' Law**, which we will look at in the optional topic, *Turning Points in Physics*. The measurement of viscosity is important for the manufacturers of confectionery and lubricating oils. Measurement of terminal speed is one technique.

Tutorial 5.07 Questions

5.07.1

An object falls vertically from a space capsule that is 10 000 m above the Moon's surface.

(a) Calculate the speed at which the object hits the surface of the moon.

Use g = 1.6 m s^{-2}.

(b) Calculate the time it takes to fall.

5.07.2

Sketch a speed time graph of a parachutist jumping from an aeroplane, reaching terminal velocity, then opening her parachute. Explain what is happening at each stage.

5.07.3

A small ball bearing is dropped into a cylinder of viscous liquid and allowed to fall until it reaches the bottom. Describe in as much detail as you can the forces that act on the ball bearing and explain why it falls at a constant velocity.

5.07.4

(a) Describe how you would measure the terminal speed of a small ball bearing in liquids of different viscosities.

(b) Suggest how the data could be used to determine viscosity.

Tutorial 5.08 Drag, Lift and Friction	
All Syllabi	
Contents	
5.081 Lift and Drag	5.082 Fluid Flow and Aeroplanes
5.083 Is drag ever useful?	5.084 Power and speed
5.085 Frictional Forces	5.086 Lubrication
5.087 Coefficient of Friction	5.088 Kinetic Friction
5.089 Is friction useful?	

Much of this tutorial is an extension, discussing the uses of drag and fiction. While you need to be aware of these concepts in general terms, you will not be expected to do quantitative analysis of these concepts. However, your tutor might well lead you on such a discussion in class.

5.081 Lift and Drag

This aeroplane is flying at a constant speed at a constant height (*Figure 94*).

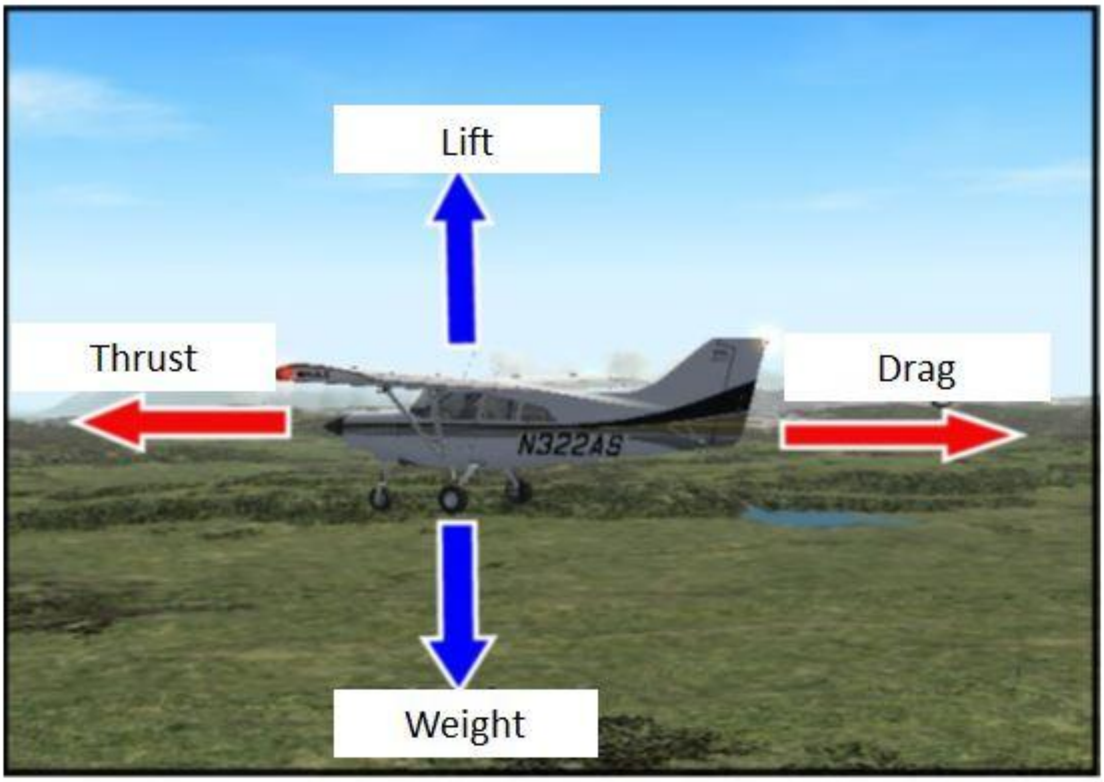


Figure 94 Forces acting on a plane.

There are two pairs of opposing forces:

- **Weight** due to gravity opposed by the **lift** from the wings.
- **Thrust** from the engine opposed by the **drag** from the air.

If the lift is equal to the weight, the aeroplane will stay at a constant height. If the thrust is equal to the drag, the speed will be constant.

Drag

Drag is a result of collisions of air molecules on the body of the aeroplane. The faster the aeroplane flies, the more frequent the collisions are, and the greater the change of momentum. We will see later that **change in momentum** results in **force**.

You can feel drag for yourself by pedalling fast on a bicycle. When you are travelling slowly, there is little air resistance. The faster you go, the greater the air resistance becomes. And that means you have to work harder.

Air is a **fluid**, which is a material that can adopt the shape of a container. In other words, a fluid is any liquid or gas.

Drag depends on these factors:

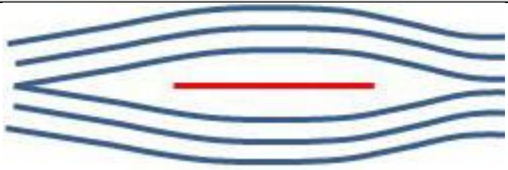
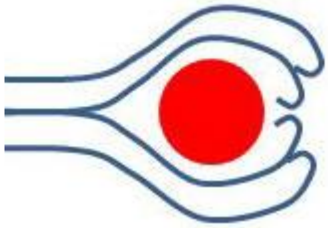
- The cross-sectional area facing the direction of movement.
- The density of the fluid.
- The square of the speed.
- The drag coefficient.

The formula (which is NOT on the syllabus) is this:

$$F_D = \frac{1}{2}\rho v^2 C_D A$$

.... *Equation 54*

You are NOT required to use this equation; it's there for interest only. The **drag coefficient** (C_D) depends on the shape of the object, and a factor called the **Reynolds number**. You can see it's getting complicated... The diagram below shows the way shape affects the drag.

Shape and Form	Form Drag / %	Skin Friction / %
	0	100
	10	90
	90	10
	100	0

You can see that a long thin shape has very low drag, but high **skin friction**. However, the skin friction is less significant in causing drag than the cross-sectional area. So, it's a good idea to make fast aeroplanes have a long cylindrical shape. This eight-oared racing shell (*Figure 95*) is long and thin. It has a low form drag, but a high skin friction.

Figure 95 An eight-oared racing shell

5.082 Fluid Flow and Aeroplanes

An old saw in the aviation world is that "if it looks good, it flies good as well".

Figure 96 Concorde (Source not known)

This aeroplane, *Concorde*, was a supersonic commercial transport aircraft that was designed in the nineteen-sixties. Even now it looks sleek and modern. However, it shared more in common with the *Vulcan* bomber (a large bomber introduced in the nineteen fifties) than with today's aircraft (*Figure 97*). Its load was limited, and it required a lot of fuel. Flying it was not easy either. The skin friction was considerable at very high speeds; its skin temperature could reach about 80 °C.

Concorde was retired in the early twenty-hundreds after a disastrous accident to one of the planes. A fuel tank on the left-hand side was struck by an object that had fallen on the runway. The fuel tanks were chock-full (which they should not have been) and the shock of the strike split the tank open, so that it was gushing out 100 litres a second. This led to a serious fire. The aeroplane was overloaded and could only just fly. Then the engineer shut down the engines on the left, resulting in the machine becoming un-flyable.

All the remaining *Concordes* are now in museums.

Figure 97 A Vulcan Bomber (Vulcan to the Skies)

This is XH558, a *Vulcan* bomber that continued to fly until September 2015. Like the Concorde, the remaining Vulcan bombers have now become valuable museum pieces.

This is a **utility aircraft** (*Figure 98*).

Figure 98 A Shorts Skyvan utility aircraft (Wiki Media Commons)

The *Shorts Skyvan* is neither elegant nor fast but can carry a large load. It is also very easy to fly.

Aeroplane design is about making compromises. Skinny long cylindrical aircraft may be fast, but they don't carry much. Stubby aeroplanes like the one above can carry good loads but are not fast.

Lift

The collision of molecules on surfaces in the right place gives rise to **lift**. Wings on aircraft are specially designed to use the **Bernoulli effect**. Air moves fast around the top of the wing and more slowly around the bottom. This causes a pressure difference (*Figure 99*).

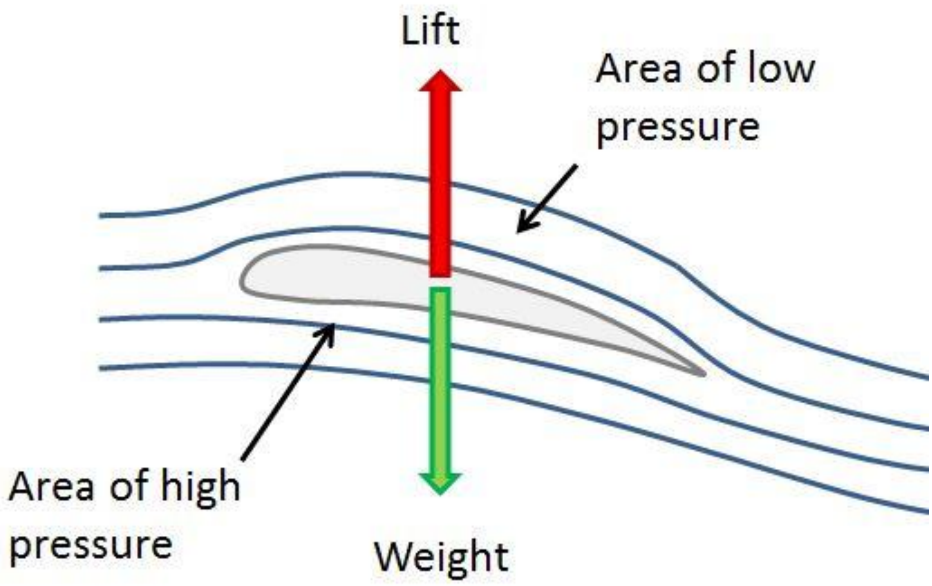


Figure 99 Lift in a wing

From GCSE you will know that:

Pressure (N m^{-2}) = force (N) ÷ area (m^{2})

$$p = \frac{F}{A}$$ .... *Equation 55*

So, if we know the pressure difference and the wing area, we can easily work out the **lift**.

Atmospheric pressure is measured in **bar** (b).

$$1 \text{ bar} = 1000 \text{ mb} = 1.013 \times 10^5 \text{ N m}^{-2}$$

Lift can be increased by increasing the angle of the wing, or by altering the shape of the wing by lowering a **flap**. Aircraft coming in to land have their flaps lowered (*Figure 100*).

- It increases the lift at low speeds.
- It also increases the drag, helping to slow the plane down.

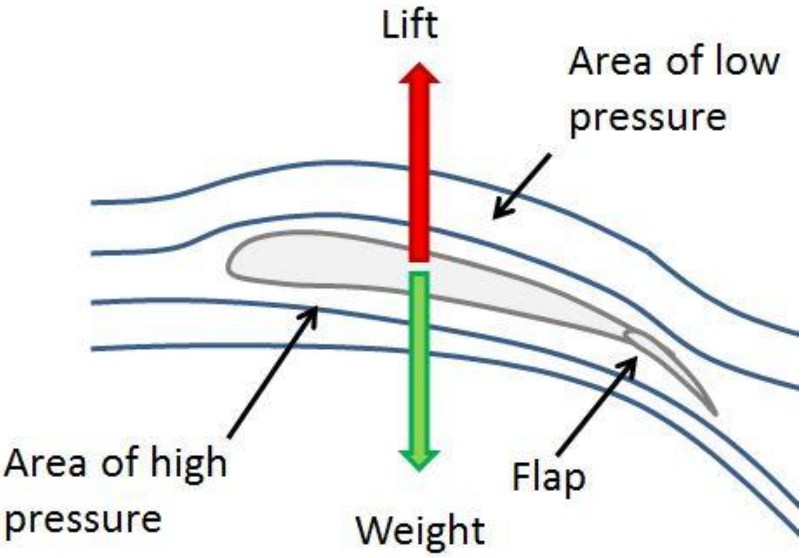


Figure 100 Lowering the flaps increases the lift.

Aircraft taking off need to **rotate**, which means that they are at an angle to the ground. This gives a higher **angle of attack**, leading to more lift. Pilots call this **alpha** because the angle is shown as α in diagrams (*Figure 101*).

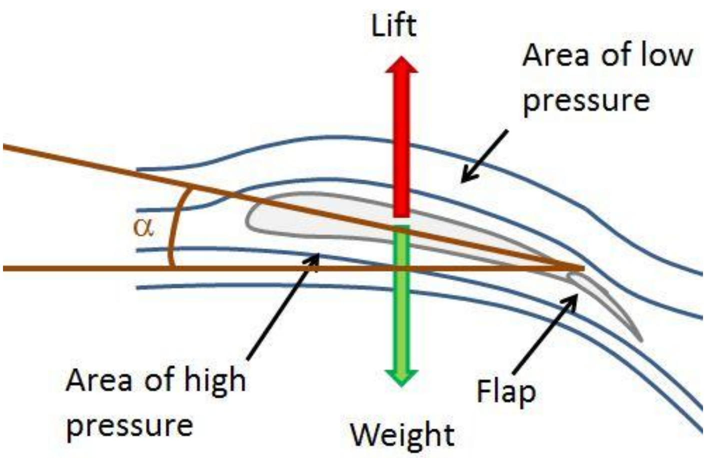


Figure 101 Angle of attack

However, if the **angle of attack** is too high, or the speed is too low, the airflow breaks up, leading to a **stall**. The plane will no longer fly and falls. At height, this can be easily recovered (and is a standard exercise with novice pilots). Near the ground it can be disastrous (*Figure 102*).

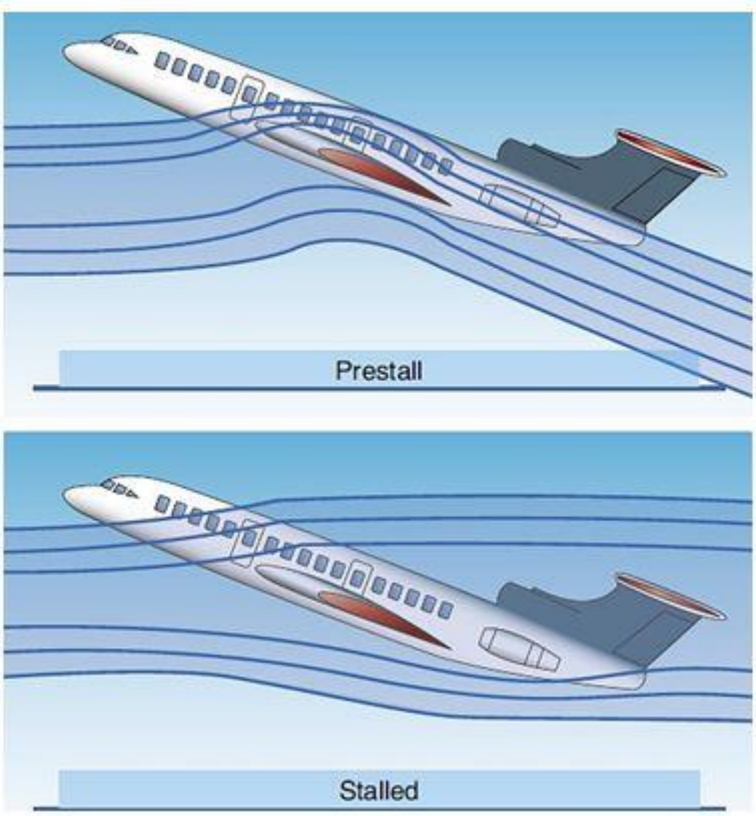


Figure 102 An aircraft stalls. (Source not known.)

In flight, the faster the aeroplane, the more the lift. This could make the aeroplane go higher, so to keep to the same level the pilot puts the nose down by **adjusting the trim**. This is done through **trim tabs** on the elevators.

5.083 Is drag ever useful?

Yes, it is. Large aircraft use spoilers to help to slow them down. These are sizeable panels that stick out into the airflow and increase drag considerably (*Figure 103*).

Figure 103 Increasing the drag (Wiki Media Commons)

Aeroplanes with high landing speeds have problems slowing down with brakes alone. So, a **braking parachute** can be deployed to slow the aeroplane down effectively (*Figure 104*).

Figure 104 A braking parachute deployed

However, this has to be repacked before the aeroplane takes off again. In the plane above, you can also see the spoiler sticking up above the cockpit.

5.084 Power and speed

Power is the rate of doing work. We have seen how power can be related to speed:

Power (W) = force (N) × speed (m s^{-1})

$$P = Fv$$ ………… *Equation 56*

We also saw how drag force was related to speed:

$$F_D = \frac{1}{2}\rho v^2 C_D A$$

……………… *Equation 57*

Again, you are NOT expected to use this equation.

Consider this light aircraft (*Figure 105*):

Figure 105 A light aircraft coming in to land

It is driven by a 1.4 litre petrol engine which gives out about 100 PS (75 kW). It flies at about 200 km h^{-1}.

Worked Example
What is the force from the propellor of this plane, if it is flying at 200 km^{-1} with a power of 75 kW?
Answer Work out the speed in m s^{-1}. v = 200000 m ÷ 3600 s = 56 m s^{-1} $P = Fv$ Therefore $F = P \div v$ F = 75000 W ÷ 56 m s^{-1} F = **1340 N**

If the aircraft above were to be made to fly at 400 km h^{-1}, the forces acting on it would be so high that it would probably break up. All aircraft have a **never exceed speed**, known to pilots as V_{NE}.

The physics of aviation is discussed in detail in **aeronautical engineering**. It can be quite complex. Aircraft design is a compromise. Very manoeuvrable aircraft are difficult to fly as they are unstable. Utility aircraft such as the Shorts Skyvan are not very manoeuvrable (they don't have to be) and are easy to fly.

5.085 Frictional Forces

Friction is a force that opposes the relative movement of solid surfaces and fluid layers. We mostly associate it with the force that opposes the movement of solid surfaces, but there are a number of different types of friction:

- **Dry friction** – the force opposing relative movement of solid surfaces.
- **Fluid friction** – the friction between layers of a fluid.
- **Lubricated friction** – the friction of a layer of lubricant between two solid layers.
- **Skin friction** – the friction that occurs on the interface between a solid and a fluid.
- **Internal friction** – the friction between layers of atoms in a solid that enables it to resist deformation.

Drag and friction are terms that are often used interchangeably. However, it is incorrect to label drag as friction, although there is the component of skin friction that contributes to drag.

Remember to write **friction**, or you will be writing *fiction*.

Dry Friction

If you look at what appears to be a smooth surface under the microscope, you will see that the surface is not at all smooth but looks more like a mountain range (*Figure 106*).

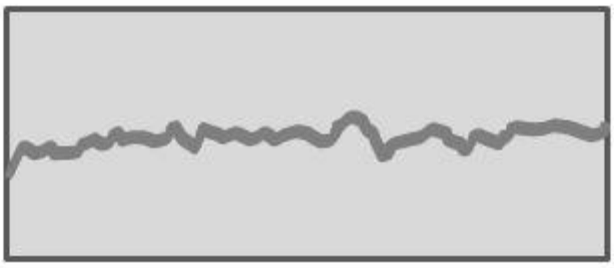


Figure 106 A smooth surface when viewed close up is anything but smooth

So, it doesn't take a genius to so that we have to do a job of work to make these two surfaces slide over each other. This wastes energy, which is why engines that have a lot of surfaces that move relative to each other are rather inefficient.

5.086 Lubrication

Dry friction can be reduced somewhat by separating the layers of solid metal by a layer of a liquid such as **lubricating** oil (*Figure 107*).

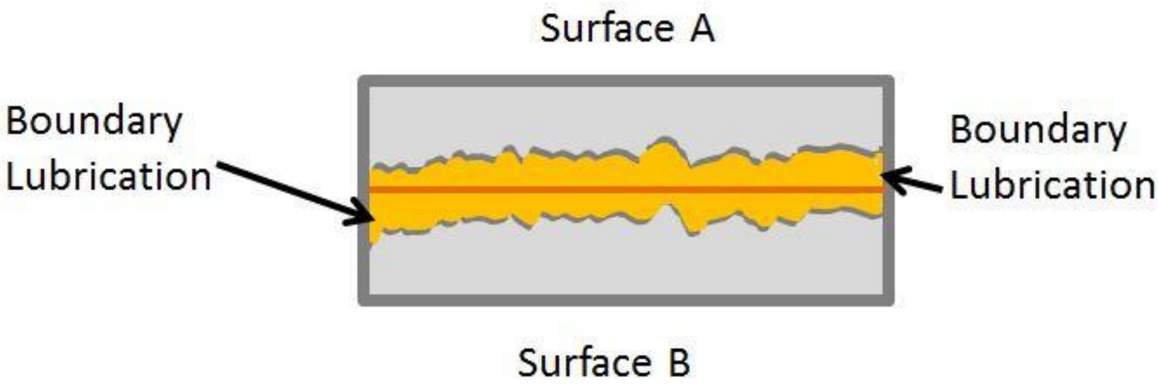


Figure 107 Lubrication causes surfaces to separate so that they can slide over each other

Lubrication does not get rid of friction completely. It reduces it. There is friction due to the lubricant itself, which is a viscous fluid.

When two surfaces are rubbing against each other, there is energy lost, which is turned into heat. If there is no lubrication, the surfaces can get so hot that they expand and increase the friction even more. In an extreme case, the surfaces can fuse together. An engine in which this happens is described as **seized up**, and the damage is expensive (*Figure 108*).

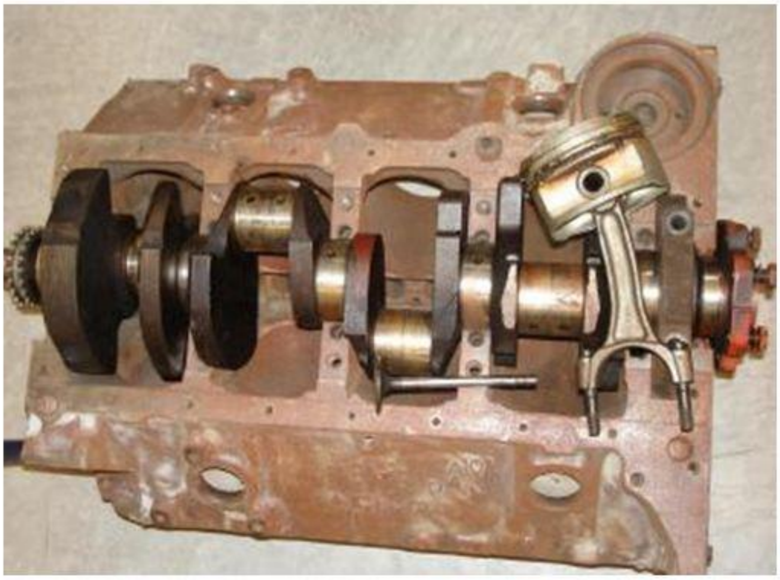


Figure 108 A seized up engine

Lubrication systems ensure that lubricant (almost always a **mineral oil**) gets to all moving parts. The lubrication system also ensures that the bearings are kept cool as well as lubricated. In some engines, there is an oil cooler, either with the main cooling system of the engine, or with the air.

5.087 Coefficient of Friction

The friction between two flat surfaces can be measured quite easily and quantified in the **coefficient of friction**. Consider a block moving on a flat horizontal surface (*Figure 109*)

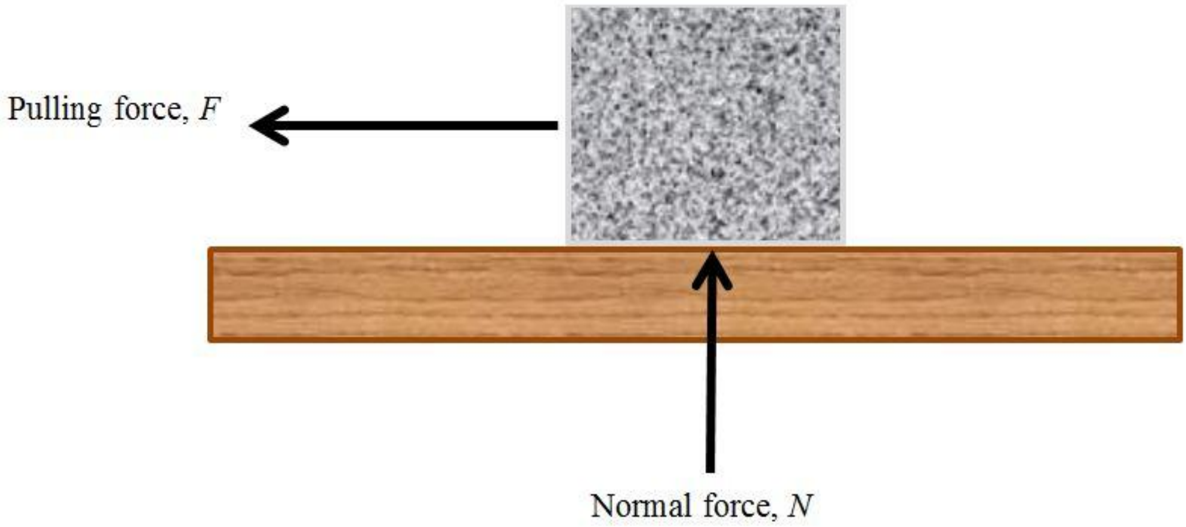


Figure 109 Coefficient of friction

In this case the **normal** force is equal (and opposite) to the weight. The coefficient of friction can be worked out using:

$$\mu = \frac{F}{N}$$

.... *Equation 58*

The term μ is "mu", a Greek letter 'm', which is the Physics code for coefficient of friction. It has no units.

If we put the block on a slope at an angle θ, the weight is no longer the normal force. The normal force is one of two perpendicular forces of which the weight is the resultant. So, we will need to work out the normal force (*Figure 110*).

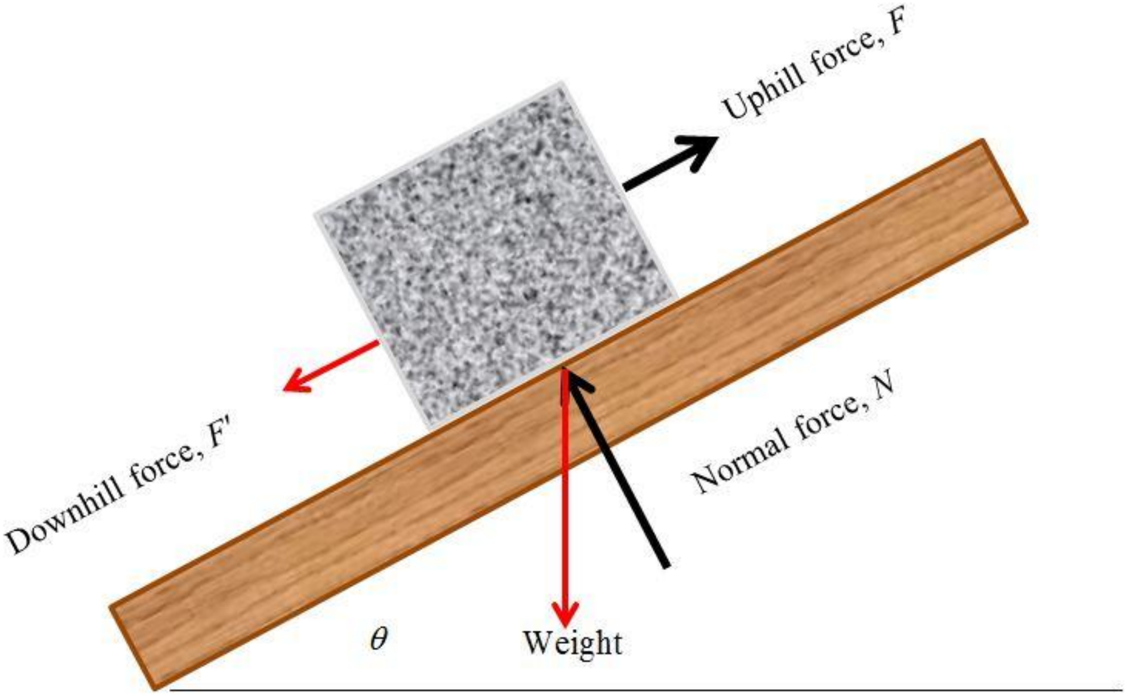


Figure 110 Coefficient of friction on a slope

From previous work, we know that:

$$W = mg$$

$$N = mg\cos\theta$$

$$F = mg\sin\theta$$

Since:

$$\mu = \frac{F}{N}$$

.... *Equation 59*

We can write:

$$\mu = \frac{mg \sin \theta}{mg \cos \theta}$$

.... *Equation 60*

It doesn't take a genius to see that weight cancels out to give us:

$$\mu = \frac{\sin \theta}{\cos \theta}$$

.... *Equation 61*

Since:

$$\tan \theta = \frac{\text{opposite}}{\text{adjacent}} = \frac{\sin \theta}{\cos \theta}$$

.... *Equation 62*

We can write:

$$\mu = \tan \theta$$

.... *Equation 63*

This result tells us:

- That the weight of an object on a slope does not affect the maximum angle to which an object can be raised to before it slides down the slope.
- The maximum angle is solely determined by the coefficient of friction.

Worked Example A slope has an angle of 25°. What is the coefficient of friction of the surface if an object just remains stationary on the slope?
Answer μ = tan 25 = **0.466** (no units)

What we have studied above is called **static friction**. It is the force needed to make the two surfaces move. **Kinetic friction** is the friction between moving surfaces. The two are different. The difference is discussed in the next section.

Here are some figures for static friction:

Material 1	***Material 2***	***Coefficient of Static Friction***
Aluminium	Steel	0.61
Cast iron	Copper	1.05
Water ice	Water ice	0.05
PTFE	PTFE	0.04
Glass	Glass	0.90
Wood	Wood	0.25 – 0.50
Rubber	Tarmac	0.9

It is possible to have a coefficient of friction greater than 1.0. It's not usual, but it simply means that the force we have to apply to get an object moving is greater than its weight. Silicone rubber against silicone rubber can have a coefficient of friction of about 1.15.

5.088 Kinetic Friction (*Pre - U and IB only*)

If you push a heavy box across a floor, you will notice that it takes more force to get it going. Once it is moving, the force needed is slightly less. When you do an experiment to measure the coefficient of friction, you will find that the force needed to make the object slide along the surface is rather less than the force needed to shift the object.

The coefficient of friction for a stationary object is for **static friction**. Its physics code is μ_s ("mu-ess"). The force that will just move a static object is given the code F_s. We know from above that:

$$F_s = \mu_s N \text{ .... } \textit{Equation 64}$$

The force required to keep the object moving at a constant speed against the friction is called the **kinetic force** (or **dynamic** force). It has the physics code F_k. The coefficient of friction in this case is the **coefficient of kinetic friction**, with the code μ_k. The equation is similar:

$$F_k = \mu_k N$$ .... *Equation 65*

Here are the figures for both static and dynamic friction:

Material 1	*Material 2*	*Coefficient of Static Friction*	*Coefficient of Kinetic Friction*
Aluminium	Steel	0.61	0.47
Cast iron	Copper	1.05	0.29
Water ice	Water ice	0.05	0.02
PTFE	PTFE	0.04	0.04
Glass	Glass	0.90	0.40
Wood	Wood	0.25 – 0.50	0.20
Rubber	Tarmac	0.9	0.50

Notice how the kinetic friction is less than the static friction.

Worked Example

A block of copper of weight 12 N is at rest on a perfectly level and smooth cast iron surface.

(a) Calculate the force that will just make the block move.

(b) The same force is applied when the object starts to move. Calculate the acceleration of the object.

(Take the value of g as 9.8 N m^{-2}.)

Answer

(a) Use:

$$F_s = \mu_s N$$

F_s = 1.05 × 12 N = **12.6 N** (= 13 N to 2 s.f.)

(b) We need to work out the kinetic force:

$$F_k = \mu_k N$$

F_k = 0.29 × 12 N = 3.48 N

Difference between the static and kinetic force = 12.6 N - 2.48 N = 9.12 N

We now need to know the mass:

m = 12 N ÷ 9.8 m s^{-2} = 1.22 kg

Now use Newton II:

$a = F/m$ = 9.12 N ÷ 1.22 kg = **7.4 m s^{-2}** (to 2 s.f)

5.089 Is friction useful?

In many cases friction is essential. The rubber feet of my laptop keep it firmly on the table as I type this stuff out. If you want to check out the usefulness of friction, try walking down an icy pavement in leather-soled shoes. You will quickly find the answer.

If rubber didn't have the high coefficient of friction, our cars would skid much more easily. With rubber, the high coefficient of friction can be explained by the idea of **mechanical keying** (*Figure 111*).

Figure 111Mechanical keying

The pliable surface of rubber is pushed into the road, increasing the area of contact. The diagram shows how the grip is improved.

Tutorial 5.08 Questions

5.08.1

How can drag be reduced?

5.08.2

Calculate the pressure exerted on the wings of a light aircraft of mass 1100 kg if the wings have an area of 16 m^2. How does this compare with atmospheric pressure which is 1.013×10^5 Pa?

5.08.3

How has the pilot increased the drag on this plane shown in *Figure 103*?

5.084

Use your understanding of kinetic energy to explain why a braking parachute is used on the military aircraft in *Figure 104*.

5.08.5

By considering the two relationships in *Equations 56* and *57*, suggest how the power is related to the speed in an aircraft.

5.08.6

A light aircraft is driven by a 1.4 litre petrol engine which gives out about 100 PS (75 kW). It flies at about 200 km h^{-1}.

Use the answer to 5.08.5 and the data above to work out the power that would be needed to make this aircraft fly at 400 km h^{-1}.

5.08.7

A piece of metal is moved on a wooden surface with a pulling force of 12.6 N. Its mass is 2.45 kg. What is the coefficient of friction?

5.08.8

A weight of 6.9 N is on a slope. When the normal force is measured, it is 6.0 N, and the object is just about to slide down the slope.

a. Calculate the angle of the slope;

b. Calculate the coefficient of friction between the material of the object and the material of the slope.

Tutorial 5.09 Projectiles

All Syllabi

Contents

5.091 Projectiles

A **projectile** is any object that is thrown or released into a gravity field. There are three situations we will look at:

1. Throwing an object vertically into the air.
2. Throwing an object horizontally from a height.
3. Throwing an object at an angle.

The **key concepts** you must understand:

- The **horizontal movement is totally independent of the vertical movement**. That means that they do NOT affect each other.
- The two movements are **vector quantities**, so they have a direction.
- The velocities are at 90° to each other.
- There is a **resultant velocity** from the two independent velocities.
- We can analyse the **vertical** movement using the equations of motion.

This topic needs you to be thinking very carefully.
Remember that the horizontal movement is **independent** from the vertical movement.

There are lots of bear traps!

5.092 General Concepts

We will call the horizontal motion the x-**component**, and the vertical motion the y-**component**. See *Figure 112*.

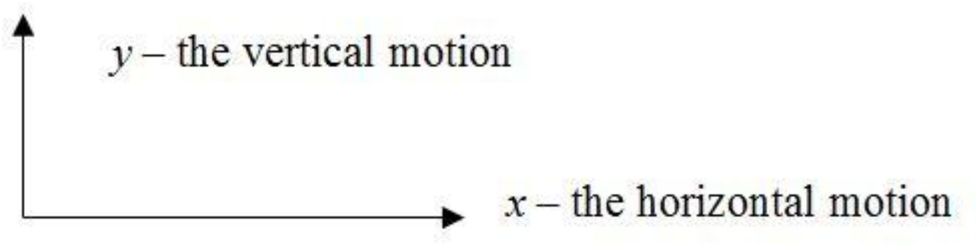


Figure 112 x and y motion of projectiles

At this level, we **always** ignore air resistance.

Let's look at the **horizontal motion**:

$$v_x = \frac{s_x}{t}$$

.... *Equation 66*

v_x and s_x are the **horizontal velocity and horizontal displacement** respectively.

You can see that it's very simple and it is affected in NO WAY by the vertical motion. If we want to know the **range** (the displacement of projectile when it lands), we need to know v_x and t.

Do not introduce the vertical velocity into your treatment of the horizontal velocity.

Now let's look at the **vertical motion**. Since we have an acceleration (g = -9.81 m s^{-2}), we use the equations of motion that involve acceleration. We also need to be aware of **the minus sign** that tells us that the acceleration is vertically **downwards**.

$$v_y = u_y + gt$$

.... *Equation 67*

$$s_y = u_y t + \frac{1}{2} g t^2$$

.... *Equation 68*

The terms u_y, v_y, and s_y stand for **initial vertical velocity**, **final vertical velocity**, and **vertical displacement** respectively.

The **directions** are important. If the vertical velocity is **positive**, it means that the object is moving **upwards**. This is also true for the displacement. Downwards is **negative**.

Whether the object is moving upwards or downwards, it is accelerating at **-9.81 m s^{-2}**.

Look at *Figure 113*.

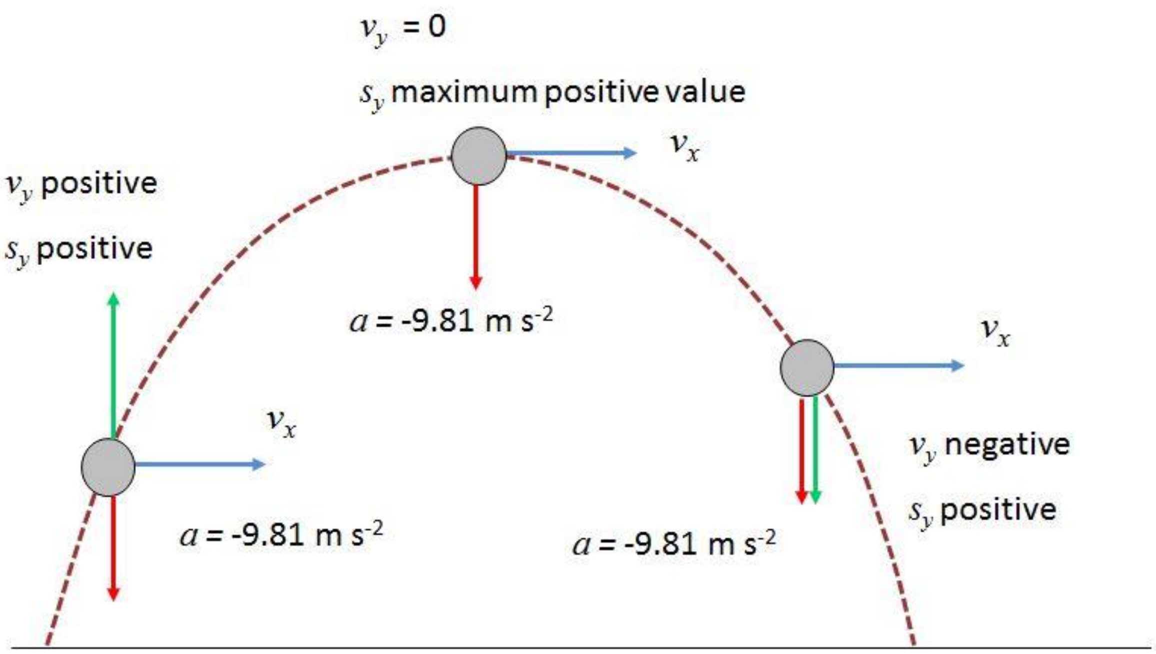


Figure 113 Directions are important in projectile motion

Some points from this diagram:

- On the ground s_y is zero.
- If the object carries on falling below the level from which it is thrown, s_y is negative.
- At the very top of the projectile path v_y is zero, but the object is **still accelerating** at –9.81 m s^{-2}.
- At all points v_x remains unchanged.
- The horizontal velocity v_x is totally independent of v_y.
- The path is a **parabola**.

Many students forget the independence of vertical and horizontal motions. They blindly put the horizontal velocity into the equations:

$$v_y = u_y + gt$$

$$s_y = u_y t + \frac{1}{2} g t^2$$

...

...usually into the u_y term. That will get zero marks in the exam.

The example below includes what NOT to do.

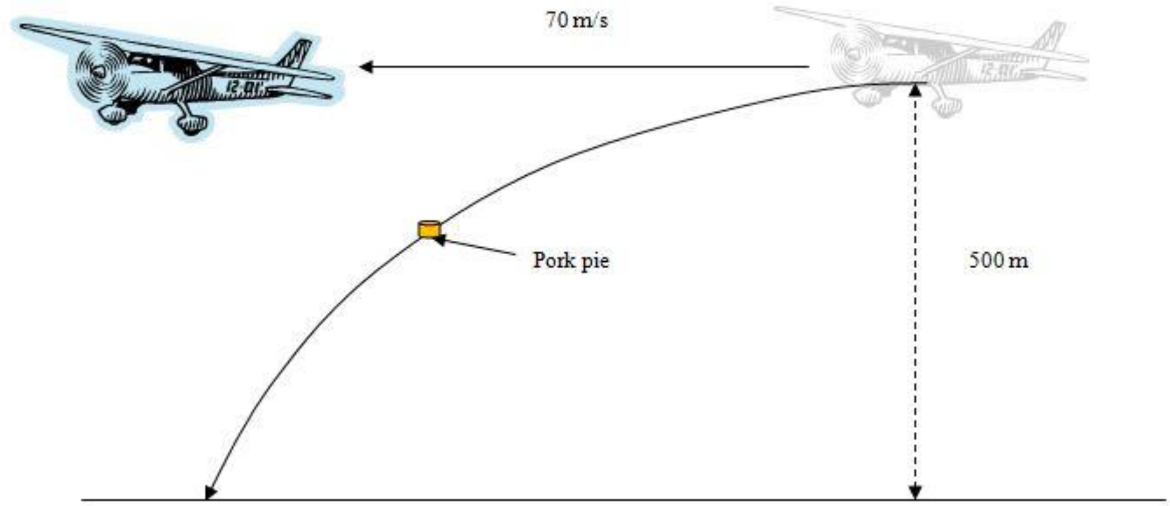
Worked Example

An aerobatic pilot is flying at a speed of 70 m s^{-1} between airshows at a height of 500 m. He is hungry and eats a pork pie which he finds utterly unpleasant. He throws it out of the aeroplane.

a) How long does the pie take to reach the ground?

b) What is the distance between the point of release and the point at which it hits the ground?

c) What is its speed when it hits the ground?



Answer

WHAT NOT TO DO

$$s_y = u_y t + \frac{1}{2} g t^2$$

Therefore:

500 m = 70 × t + ½ × -9.8 m/s^{-2} × t^2

The student has put the horizontal velocity into the vertical equation of motion.

Please, be a good chap and don't do this.

Now the correct way:

- Horizontal velocity = 70 m s^{-1}
- Vertical displacement = -500 m (it's downwards)
- Initial vertical velocity = 0 m s^{-1}

a) Use

$$-500\text{ m} = 0 \times t + \tfrac{1}{2} \times -9.81\text{ m s}^{-2} \times t^2$$

$$t^2 = (-500\text{ m} \times 2) \div -9.81\text{ m s}^{-2}$$

$$t^2 = 1000\text{ m} \div 9.81\text{ m s}^{-2} = 102\text{ s}^2$$

$$t = \sqrt{102\text{ s}^2} = \underline{\mathbf{10.1\ s}}$$

Note how the displacement is negative, and the acceleration is negative, so the minus signs cancel out. Time is always positive; you cannot go back in time.

b) Use:

$$v_x = \frac{s_x}{t}$$

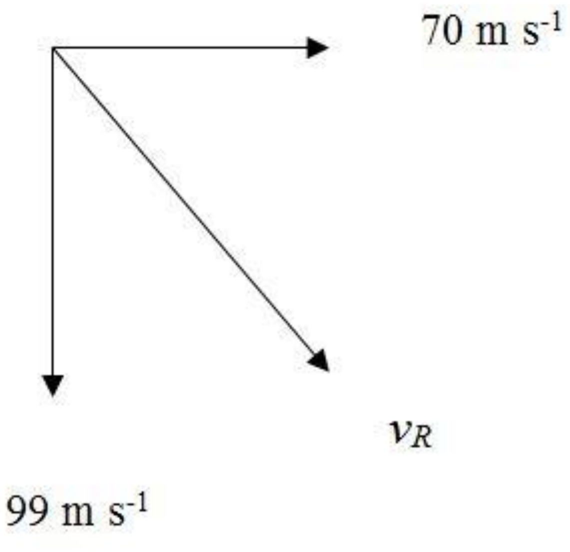
$$s_x = 70\text{ m s}^{-1} \times 10.1\text{ s} = \underline{\mathbf{707\ m}}$$

c) Work out the vertical velocity using:

$$v_y^2 = 0 + 2 \times -9.81 \text{ m s}^{-2} \times -500 \text{ m} = 9810 \text{ m}^2 \text{ s}^{-2}$$

$$v_y = 99 \text{ m s}^{-1}$$

Now we can work out the resultant velocity:



Work out v_R:

$$v_R^2 = (99 \text{ m s}^{-1})^2 + (70 \text{ m s}^{-1})^2$$

$$v_R^2 = 9810 \text{ m}^2 \text{ s}^{-2} + 4900 \text{ m}^2 \text{ s}^{-2} = 14710 \text{ m}^2 \text{ s}^{-2}$$

v_R = **<u>121 m s^{-1}</u>**

Projectile problems often cause problems for students. They are not that difficult as long as you remember the problem-solving strategy. Let us look at three different situations.

5.093 Throwing An Object Vertically Into The Air

Consider a basketball player throwing a ball in the air. What goes up must come down.

Figure 114 Throwing a ball vertically into the air (Picture from a clip art collection)

The ball has a downward force acting on it because of gravity. Therefore, it will slow down at a rate of 9.81 m s^{-2}. So, we can say that the acceleration is -9.81 m s^{-2}. When we tackle problems like this, we use the equations of motion. Go to Tutorial 5.06 if you need to review the equations of motion. We have to make sure that we get the signs right. We will make **upwards positive** and **downwards negative**.

We can represent these motions graphically. It is important that you understand these graphs.

A displacement time graph looks like this (*Figure 115*).

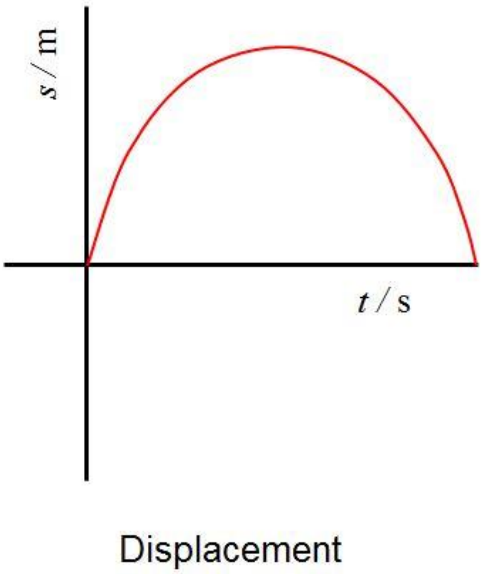


Figure 115 Displacement-time graph for a ball thrown vertically in the air.

The graph is a parabola because the ball is accelerating downwards. When it reaches the top, its velocity is 0, but its acceleration is still (-)9.81 m s^{-2}.

The velocity time graph looks like this (*Figure 116*).

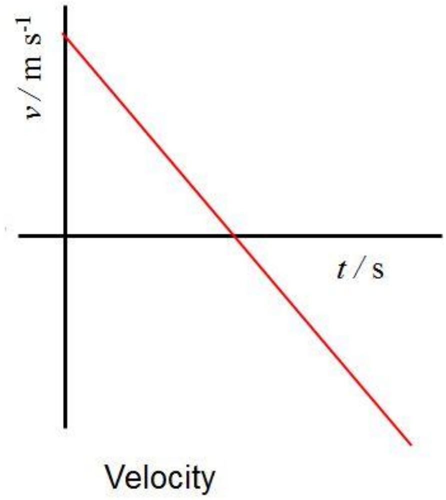


Figure 116 Velocity-time graph for a ball thrown vertically in the air.

This graph shows how the velocity not only changes, but its sign changes as well. This tells us that the direction changes as well. This stands to reason; if going up is positive, going down must be negative. Note that the gradient is constant, i.e. the acceleration is constant. The acceleration time graph looks like this (*Figure 117*).

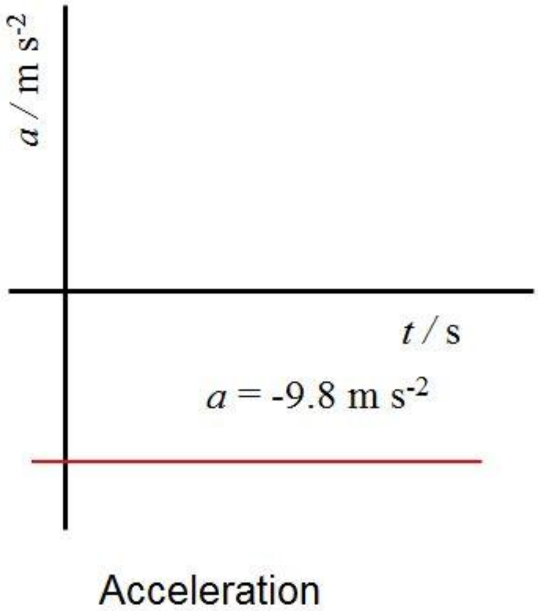


Figure 117 Acceleration-time graph for a ball thrown vertically in the air.

It shows us that the acceleration is constant at -9.81 m s^{-2}. The minus sign tells us that the acceleration is towards the ground.

5.094 Throwing an object horizontally from a height

If we throw an object horizontally, there are two important things to consider:

1. The horizontal velocity remains constant (ignoring air resistance)
2. The vertical velocity increases at a rate of (-)9.8 m s^{-2}.

If we throw one object horizontally and drop a second object vertically at the same time, we see this (*Figure 118*). This is easier said than done!

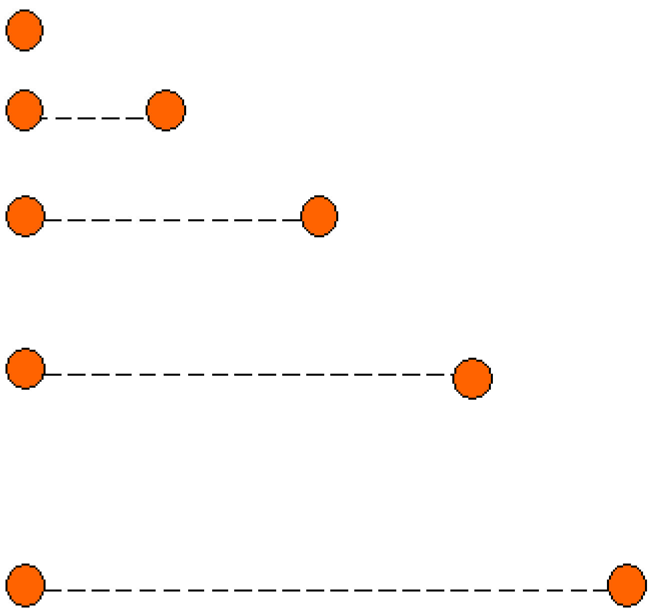


Figure 118 Throwing two objects

The second object, thrown horizontally, will hit the ground at the same time as the object that is simply dropped. Although the drawing is not to scale, you can see how the horizontal velocity remains constant, while there is acceleration downwards.

Look at the diagram below. A pallet is dropped from a helicopter to the ground. We will ignore the air resistance (*Figure 119*).

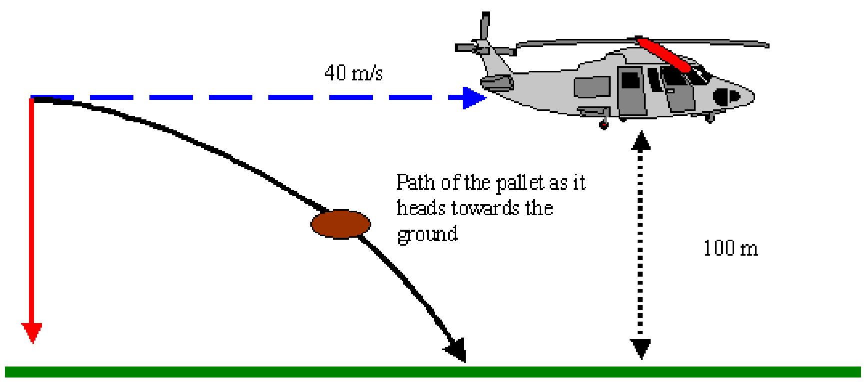


Figure 119 A helicopter drops a pallet of aid to the ground

The path taken is NOT a straight line, because the velocity downwards is increasing at a constant rate of 9.8 m/s^2. It is a **parabola**. There are two components in this problem:

- The **horizontal velocity** which remains **constant.**
- The **vertical velocity** which changes, as the object is accelerating towards the ground. We use an **equation of motion** to analyse the motion.

The key point to remember is that the horizontal and the vertical motions are independent.

A common bear-trap is to put the horizontal velocity into the vertical equation of motion.

5.095 Throwing an object at an angle

Figure 120 Archery practice (Picture by Casito, Wikimedia Commons)

Archery is a sport in which the participants subconsciously do calculations involving movement in two directions. Again, the **vertical and horizontal movement are independent**.

Let us analyse the motion from the moment an arrow is released to the moment it hits the target. We want to find the range. For simplicity we will assume that the target is at the same height as the release point. We will also ignore air resistance. We will not worry about the signs (*Figure 121*).

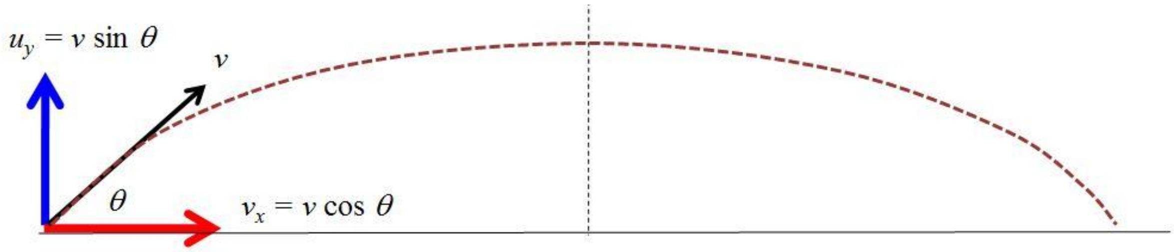


Figure 121 Trajectory of an archer's arrow

The diagram above shows the trajectory of the arrow. It shows the movement in space. It is NOT a motion-time graph!

1. On release, the arrow leaves at a velocity v m s^{-1} and angle θ. The horizontal velocity is $v \cos \theta$ m s^{-1}. The horizontal velocity remains constant at $v \cos \theta$ m s^{-1}(*Figure 122*).

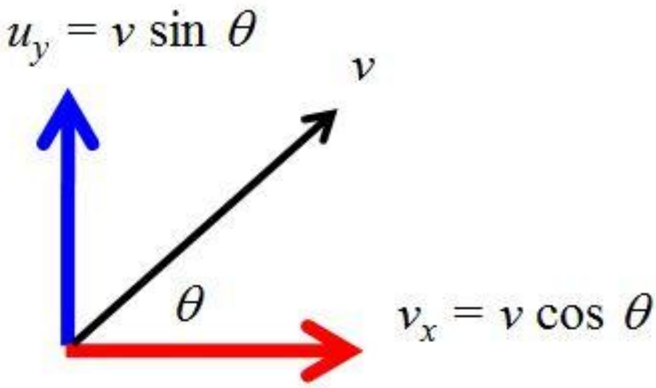


Figure 122 Resolving the components of the initial velocity when the arrow is released

2. The vertical velocity is $v \sin \theta$ m s^{-1} initially.

3. To work out the time we need to use an equation of motion that has initial velocity, acceleration, and time.

The equation:

$$v = u + at$$ .... *Equation 69*

will fit the bill. As a is negative, *Equation 69* becomes:

$$0 = v \sin \theta + -at$$

$$v \sin \theta = at$$ .... *Equation 70*

Therefore

$$t = \frac{v \sin \theta}{a}$$

.... *Equation 71*

4. To get the range we need to multiply the horizontal velocity by the time taken in the air. Therefore:

$$\text{range} = v \cos \theta \times 2t$$ .... *Equation 72*

It is $2t$ because it takes t seconds for the arrow to go up to its maximum height and t seconds for it to come down again.

Worked example A large firework rocket leaves a launch tube at a velocity of 110 m s^{-1} at an angle of 30 degrees. What is the range of the rocket?
Answer 1. Work out the horizontal velocity. Horizontal velocity = $v \cos \theta$ = 110 m s^{-1} x cos 30 = 110 m s^{-1} x 0.866 = **95.3 m s^{-1}**
2. Work out the initial vertical velocity: The initial vertical velocity = $v \sin \theta$ = 110 m s^{-1} sin 30 = 110 m s^{-1} x 0.5 = **55 m s^{-1}**
3. Now work out the time it takes to get to the maximum height: 0 = 55 m s^{-1} + -9.81 m s^{-2} x t ($\Rightarrow$ 0 - 55 m s^{-1} = -9.81 m s^{-2} × t) t = 55 m s^{-1} ÷ 9.81 m s^{-2} = 5.61 s Therefore, the total time in the air = 2 x 5.61 s = **11.2 s**
4. Therefore, the range = $v \cos \theta \times 2t$ = 11.2 s × 95.3 m s^{-1} = **1070 m.**

In the First Year (AS) exam, they will not be over officious with signs, but make sure you explain each step. However, it's good practice to get the signs right. In the A level exam, they will be expecting correct use of the signs for full marks.

5.096 Finding the range without the time

Consider an arrow fired at an angle θ at a velocity of v m s^{-1}. The trajectory is like this (*Figure 123*).

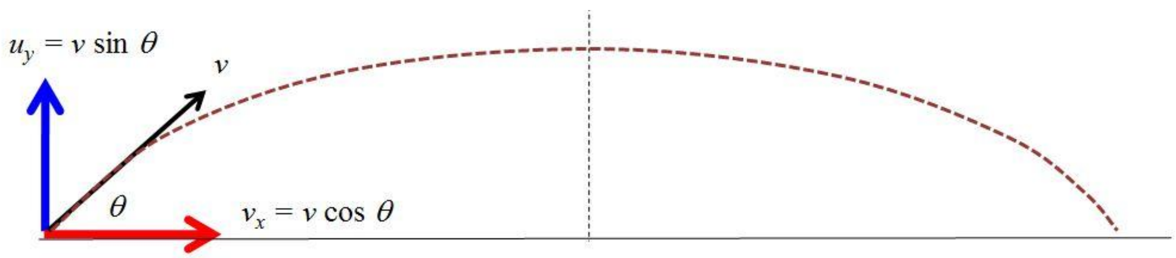


Figure 123 Trajectory of a projectile fired into the air

We know that the horizontal velocity remains constant.

$$v_x = v \cos \theta$$ .... *Equation 73*

We know that the vertical velocity changes. The vertical velocity at the start is:

$$v_y = v \sin \theta$$ .... *Equation 74*

So, we can write:

$$u_y = v \sin \theta$$ .... *Equation 75*

The vertical velocity is 0 at the top of the trajectory. Work out time taken to get to top:

$$0 = u_y + gt$$

.... *Equation 76*

Therefore

$$t = \frac{-u_y}{g}$$

.... *Equation 77*

Therefore, we can write:

$$t = \frac{-v \sin \theta}{g}$$

.... *Equation 78*

The range is s_x, and the time taken is $2t$, because t is the time for the arrow to get to the top. What goes up must come down, and it takes exactly the same time to fall as it does to go up. Therefore, we can write:

$$s_x = 2t \times v_x = 2t \times v \cos \theta$$ .... *Equation 79*

Since:

$$t = \frac{-v \sin\theta}{g}$$

.... *Equation 80*

We can substitute *Equation 80* into *Equation 79* to get:

$$s_x = 2v\cos\theta\left(\frac{-v\sin\theta}{g}\right)$$

.... *Equation 81*

Note that g = - 9.81 m s^{-2}, so the minus signs cancel out. The time is always positive (you can't go back in time).

Worked Example A large firework rocket leaves a launch tube at a velocity of 110 m s^{-1} at an angle of 30 degrees. What is the range of the rocket?
Answer Use: $s_x = 2v\cos\theta\left(\frac{-v\sin\theta}{g}\right)$ Put in the numbers: $s_x = (2 \times 110 \text{ m s}^{-1} \cos 30) \times \left(\frac{-110 \text{ m s}^{-1} \sin 30}{-9.81 \text{ m s}^{-2}}\right)$ $s_x = (190.5 \text{ m s}^{-1} \times \left(\frac{-55.0 \text{ m s}^{-1}}{-9.81 \text{ m s}^{-2}}\right) = 1068 \text{ m}$ The range is 1070 m to 3 s.f. This is consistent with the answer in the previous worked example.

Tutorial 5.09 Questions

5.09.1

The girl throws the ball at an upward velocity of 15 m s^{-1}. How high will it go?

5.09.2

How long will it take the ball to reach its maximum height?

In both questions use g = 9.8 m s^{-2}

5.09.3

What would the **speed time graph** look like for a call thrown vertically in the air?

Refer to *Figure 119* for Questions 5.09.4 to 5.09.6.

5.09.4

What is the horizontal velocity?

5.09.5

Show that the **vertical velocity** about 44 m s^{-1} towards the ground? Note that the horizontal velocity is ignored. Use g = 9.8 m s^{-2}

5.09.6

What is the resultant velocity of the pallet just before it hits the ground?

5.09.7

A javelin thrower throws a javelin at a velocity of 25 m s^{-1} at an angle of 40 degrees. What distance will he throw the javelin?

Use g = 9.8 m s^{-2}

Tutorial 5.10 Newton's Laws of Motion	
All Syllabi	
Contents	
5.101 Newton I	5.102 Newton II
5.103 Newton III	5.104 Inertia

5.101 Newton I

Newton's First Law states:

Every object continues in its state of rest or uniform motion in a straight line, unless it is compelled to change that state by an external force acting on it. (*Figure 124*)

Figure 124 A car travelling at constant speed obeys Newton' First Law

A car will maintain a constant speed if the drive force and the drag are **balanced**. **The total force is zero**.

5.102 Newton II

Newton's Second Law states:

Rate of change of momentum is proportional to the total force acting on a body and occurs in the direction of the force.

An object of mass, m, is acted on with a constant force, F, so that its velocity increases from an initial value, u, to a final value, v, in time, t.

Momentum is the product of mass and velocity. It is a vector and has units of kilogram metres per second (kg m s^{-1}).

$$\frac{\text{Change in Momentum}}{\text{Time}} \propto \text{Force}$$

.... *Equation 82*

Change in momentum = momentum at end - momentum at start

Therefore:

$$\frac{mv - mu}{t} \propto F$$

.... *Equation 83*

Therefore

$$\frac{m(v - u)}{t} \propto F$$

.... *Equation 84*

Now we know that acceleration:

$$a = \frac{v - u}{t}$$

.... *Equation 85*

So, we can write:

$$F \propto ma$$

.... *Equation 86*

Therefore:

$$F = kma$$

.... *Equation 87*

(The term *k* is a constant with no units. $k = 1$)

Force (N) = Mass (kg) × acceleration (m s^{-2})

$$F = ma \text{ .... } \textit{Equation 88}$$

Make sure you use the right units:

- **Force** in **N**,
- **Mass** in **kg**,
- **Acceleration** in **m s^{-2}**.

Acceleration is always caused by a **total force**, or the **resultant force**, the **vector sum** of all the forces. The acceleration is always, without exception, in the **same direction as the total force**.

Strictly speaking, we should write the equation as:

$$\Sigma F = ma \text{ .... } \textit{Equation 89}$$

The strange symbol Σ is Sigma, a Greek capital letter 'S'. It is physics code for "sum of". Consider two coplanar forces F_1 and F_2 acting at right angles. To get ΣF, we need to do a vector sum on the two forces (*Figure 125*).

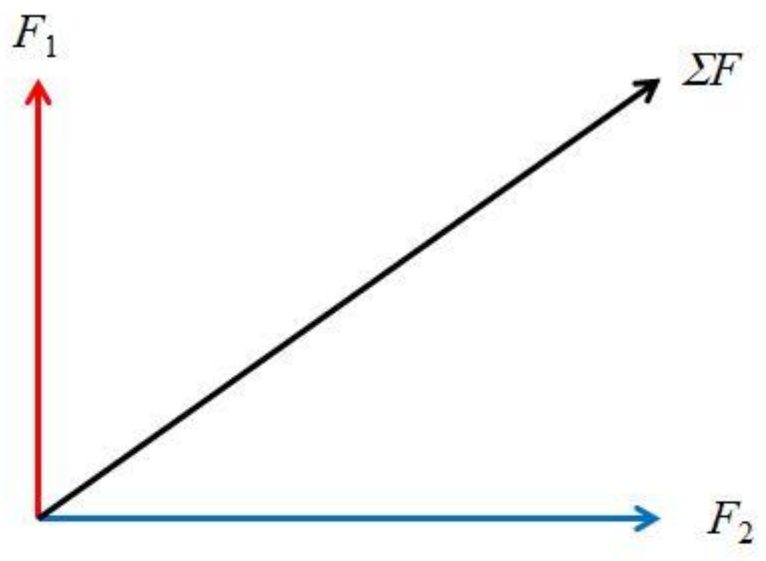


Figure 125 Resultant of two forces, ΣF

If you have a vehicle providing a force to accelerate another vehicle, you must add the mass of the towing vehicle to that being towed.

5.103 Newton III

Newton's Third law states that:

If body A exerts a force on body B, body B must exert an equal and opposite force on body A.

In other words, **forces always act in pairs**. This is true whether the forces are in equilibrium, moving, stationary or accelerating.

Figure 126 A simple hovercraft (Source not known)

This boy in *Figure 126* is sitting on a simple hovercraft (Yes, he's probably a father of teenage kids by now!). The motor drives a fan which forces air downwards onto the floor. The force of the air going down produces a reaction force that lifts the machine off the ground. A simplified diagram of the machine is shown below (*Figure 127*).

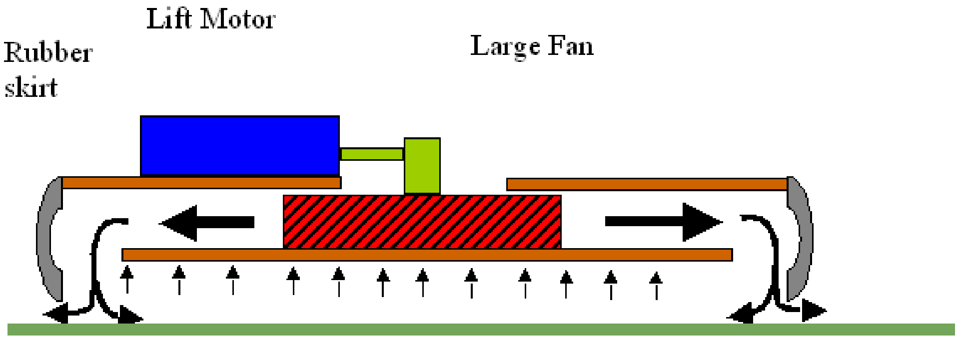


Figure 127 A diagram of a simple hovercraft

This picture (*Figure 128*) shows a small hovercraft that can skim on water or dry land. It is powered by a small (10 kW) petrol motor.

Figure 128 A small hovercraft

Hover mowers are very simple small hovercraft. Some transport hovercraft are enormous machines of mass several hundred tonnes. They move easily across flat ground. The problem with hovercraft is that their performance on hills is hopeless. Can you think why?

Figure 129 A large transport hovercraft (Picture by Andrew Berridge, Wikimedia Commons)

Newton I and III are often confused.

5.104 Inertia (Extension only)

Let's go back to Newton I:

Every object continues in its state of rest or uniform motion in a straight line, unless it is compelled to change that state by an external force acting on it.

Now consider a space probe in deep space moving at 50 km s^{-1}. The sum of all the gravity acting on the probe is zero. There is no force from the probe's motors. There are no other forces to act on the probe. However, the probe has a constant velocity of 50 km s^{-1}.

It may sound counter-intuitive that the probe keeps on going at a constant speed. It should slow down and stop. That happens here on Earth. if we look at the second part of Newton I, the key part is, "**unless it is compelled to change that state by an external force acting on it**". The external forces acting on an object here on the ground include friction and drag. These are not present in space.

This leads us to the concept of **inertia**, which is the **resistance of an object to change in its state of motion**.

Consider this situation (*Figure 130*).

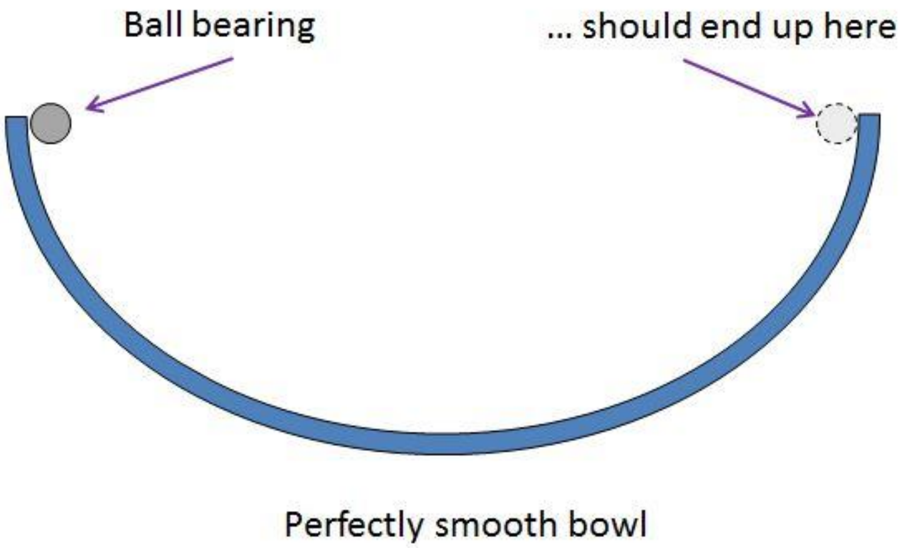


Figure 130 Releasing a ball bearing in a completely smooth bowl

A ball bearing is released on the left-hand side to run down the side of a perfectly smooth bowl. If there is no friction or drag, the ball bearing should reach the equivalent spot on the other side, and the height from the bottom will be exactly the same. This shows that the ball bearing has inertia, i.e. resistance to change in motion.

We know that all the energy has been converted from gravitational potential to kinetic, and back to gravitational potential. Therefore:

$$E_p = E_k \quad \text{………….} \quad \textit{Equation 90}$$

$$mg\Delta h = 1/2\ mv^2 \quad \text{…………} \quad \textit{Equation 91}$$

Normally we would cancel out the mass on each side. But the mass is involved in the transfer of the potential to kinetic energy and back. Remember that the moving bearing was not subject to any external forces, so the quantity involved was the inertia. So we can say the inertia has something to do with the mass. And this is the case:

Mass is the quantity that solely depends on the inertia.

The more inertia a mass has, the greater the mass. We then can say that mass is the **amount by which an object resists change of motion**. Strictly speaking we should call this mass **inertial mass**. Gravitational mass is mass involved with gravity fields. Experiments show that both are identical.

You may find this video-clip helpful:

https://www.youtube.com/watch?v=1kjgVcflx0Y

To sum up:

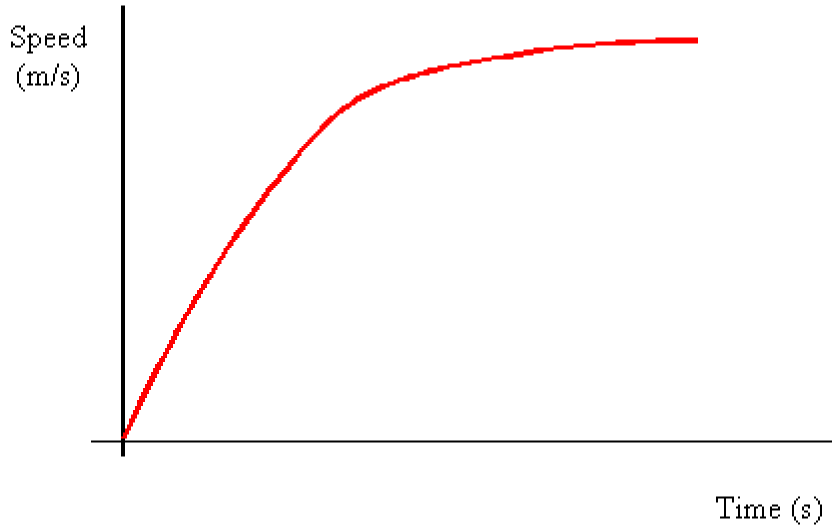
- Mass is a measurable quantity, while inertia is a concept that that explains how hard it is to change the current state of mass.
- In classical physics, mass is a property of an object, while inertia is a property of motion of the object, as well as the mass.
- Inertia is a concept that describes mass.

All of this may seem to be pedantic, and inertia is one of those topics that physicists get really wound up with. On any physics forum, there is a range of views from a variety of contributors. Many physicists go back to the idea that mass is "the amount of material in an object", which is perfectly good for most purposes. However, in rotational mechanics, inertia in its form of **moment of inertia** is an important concept to explain the behaviour of spinning objects.

Tutorial 5.10 Questions

5.10.1

The graph below shows the acceleration of a car up to its maximum speed.



(a) Why is the graph a straight line at low speeds?

(b) Use Newton I to explain why the car reaches a maximum speed which it cannot exceed. Assume it's on a test track, so it can exceed the 115 km h^{-1} national speed limit.

5.10.2

A 70 kg athlete accelerates to his maximum speed of 9.5 m s^{-1} in a time of 2.5 s. What is the average force he applies to the track?

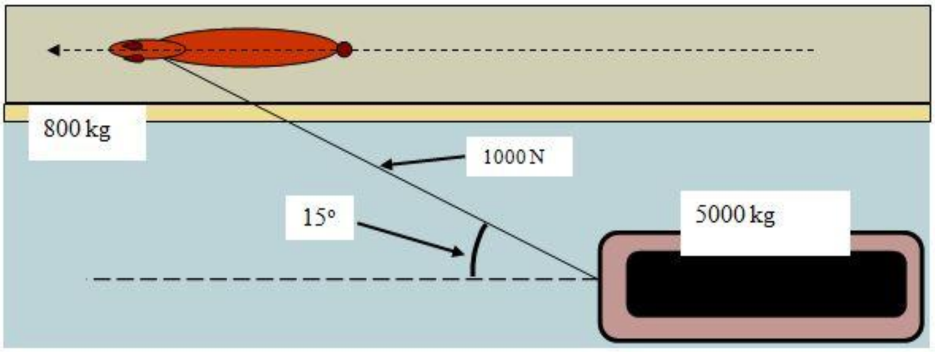
5.10.3

A locomotive of mass 100 tonnes is hauling a train of wagons of mass 1200 tonnes with a pulling force (tractive effort) of 180 kN. What is the acceleration of the train?

(1 tonne = 1000 kg)

5.10.4

A horse of mass 800 kg is pulling a barge of mass 5000 kg as shown in the diagram below:



The barge is initially at rest.

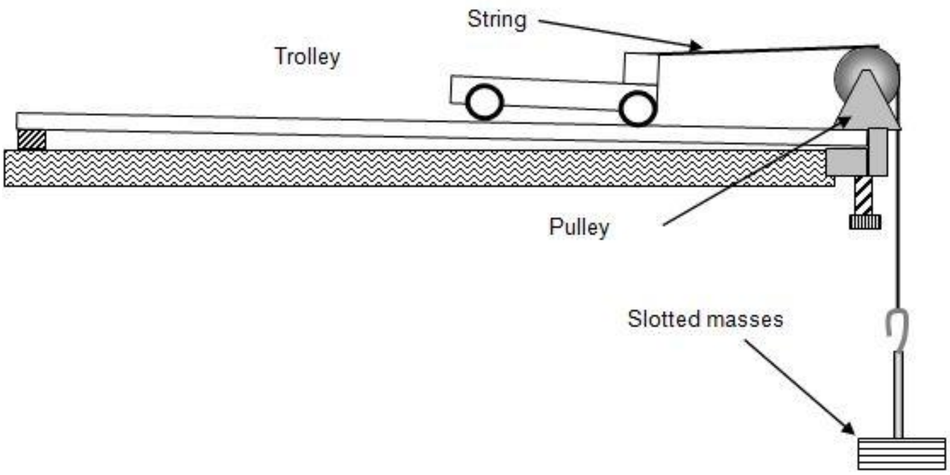
(a) Calculate the acceleration.

(b) The horse reaches a maximum speed of 1.5 m s^{-1}. Calculate the time taken to reach this speed.

5.10.5

(Challenge) This is part of an examination question

A trolley of mass 1.0 kg is set up on a friction compensated runway, and attached to a 1 N slotted mass, which is released.



The trolley moves down the runway, and its motion is recorded with a ticker tape timer. Further slotted masses are added, and the motion measured. Describe how this experiment can be used to verify Newton's Second Law of Motion.

Explain why this is a bad question.

Tutorial 5.11 Momentum and Impulse	
All Syllabi	
Contents	

5.111 Momentum Basics

Momentum is the **product between mass and velocity**. Being a **vector** quantity, it has a **direction**, and the direction is very important when doing momentum calculations. Momentum is not a thing that we can see, (I cannot order a class set of momentums from the physics technician) but it does explain many things that go on in physics.

Momentum (kg m/s) **= mass** (kg) x **velocity** (m/s)

$$p = mv$$ .... *Equation j92*

Units are **kilogram metres per second** (kg m/s) or **Newton seconds** (N s). We can show that the units are the same.

1 N = 1 kg m s^{-2} (Newton II)

1 kg m s^{-1} = 1 kg m s^{-2} × s = 1 N s

If you answer question 5.11.1, you will consider only the *value* of the momentum. This is why we used the word *speed*. It is very important to make sure that you pay attention to the signs when doing momentum calculations.

Think of a ball bouncing off a wall. It leaves the wall at the same speed as before. Let's call going from **right to left** <u>negative</u>, and going from **left to right** positive (*Figure 131*).

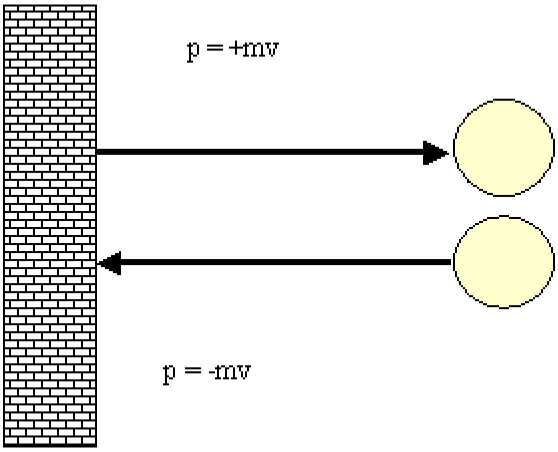


Figure 131 A ball bouncing off a wall

5.112 Impulse

The **change in momentum** is called the **impulse** and is given the physics code Δp. We can define Newton's Second Law in terms of change of momentum:

Force = mass × acceleration

Force = mass × $\dfrac{\text{change in velocity}}{\text{time interval}}$

Change in velocity = velocity at end - velocity at start

Change in momentum = mass × change in velocity

Therefore:

Force (N) = $\dfrac{\text{change in momentum (Ns)}}{\text{time interval (s)}}$

$$F = \frac{\Delta p}{\Delta t}$$

.... *Equation 93*

$\Rightarrow$ **Impulse** (Ns) = **Force** (N) x **time interval** (s)

In code:

$$\Delta p = F\Delta t$$

.... *Equation 94*

If we plot a force time graph, we can see that impulse is the **area under the graph** (*Figure 132*).

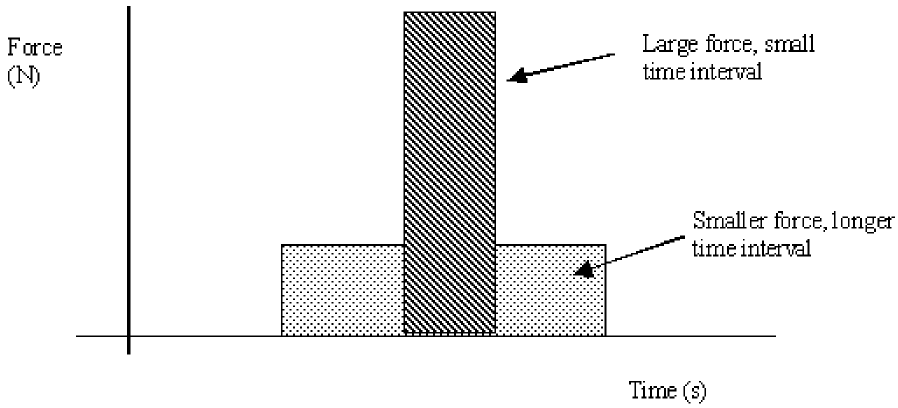


Figure 132 Force-time graph showing impulse.

In this graph, both impulses are the same. The forces and time intervals are different. In these cases, the force is constant.

The graph below (*Figure 133*) shows the effect of a force that is not constant.

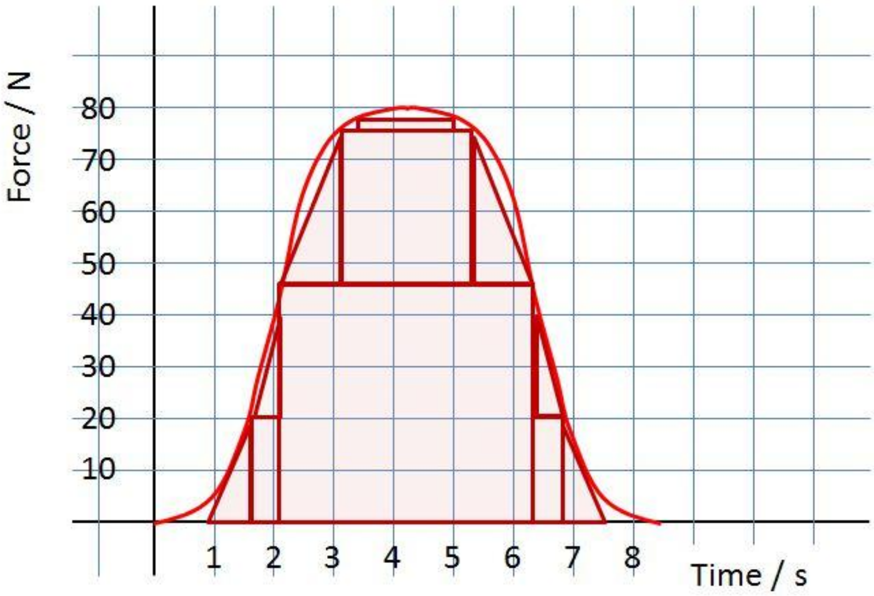


Figure 133 Force-time graph of impulse in which the force is not constant

We can work out the impulse to a first approximation by working out the areas of the rectangles and triangles and adding them together. In calculus notation, we can write:

$$\Delta p = \int F \, \mathrm{d}t$$

.... *Equation 95*

If the force is varying to a known function, we can work out the area under the graph using integration. You will NOT be expected to do this at this level.

5.113 Impulse and Newton's Second Law

Consider an object of mass m which is subject to a force F for a time period of t seconds. We can say that there is an impulse on the object:

$$F = \Delta p/\Delta t = \Delta mv/\Delta t = m\Delta v/\Delta t$$ .... *Equation 96*

The term $\Delta v/\Delta t$ = change in velocity ÷ time interval = acceleration

So, **Force = mass × acceleration** which is Newton II. Remember that wherever there is a resultant force, there is always acceleration. A change in momentum results in acceleration, which is caused by a force. We saw another version of this in the previous tutorial.

Impulse is the physics phenomenon that explains how a ball behaves when kicked or hit with a bat. It also has important implications in road safety. When a car is involved in a collision, we want the impulse to occur over a longer time interval to reduce the forces involved.

5.114 Kinetic Energy and Momentum (Edexcel Syllabus)

We know that for a mass m kg travelling at a velocity v m s^{-1}, the momentum p kg m s^{-1} is given by:

$$p = mv \quad \text{………… Equation 97}$$

We also know that for kinetic energy:

$$E_k = \frac{1}{2}mv^2 \quad \text{………… Equation 98}$$

We transpose the momentum equation (*Equation 97*) for v:

$$v = \frac{p}{m} \quad \text{………… Equation 99}$$

We can now substitute this (*Equation 99*) into the kinetic energy equation:

$$E_\mathrm{k} = \frac{1}{2}m\frac{p^2}{m^2} \quad \text{………… Equation 100}$$

Tidying this up gives us:

$$E_\mathrm{k} = \frac{p^2}{2m} \quad \text{………… Equation 101}$$

Tutorial 5.11 Questions

5.11.1

What is the value of the momentum of a 10 kg ball running down a bowling alley at a speed of 5 m s^{-1}?

Refer to *Figure 131* in answering 5.11.2 and 5.11.3.

5.11.2

Show that the change in momentum is $+2\ mv$.

5.11.3

The ball has a mass of 200 g, The *value* of its velocity throughout remains 6 m s^{-1}. What is the change in momentum?

5.11.4

Explain how the formula $F = \Delta p/\Delta t$ is consistent with Newton II.

5.11.5

A car is involved in a collision in which it is brought to a standstill from a speed of 24 m s^{-1}. The driver of mass 85 kg is brought to rest by his seat belt in a time of 400 ms.

a) Calculate the average force exerted on the driver by his seat belt.

b) Compare this force to his weight and hence work out the "g-force"

c) Comment on the likelihood of serious injury.

5.11.6

Use the change in momentum of the driver in Question 5, and his mass to calculate the work done on him.

Tutorial 5.12 Conservation of Momentum

All Syllabi

Contents

This is quite a long tutorial. You may find it easier to work through it in several sessions.

Section 5.125 is for A level only. Section 5.126 is for the OCR syllabus only.

There is an extension (5.128) which is challenging, suited to the A or Pre-U student.*

We have seen how momentum is the **product between mass and velocity**. We cannot see a momentum. There is no school or college (or university) that has a class set of momenta (plural of momentum). But it is a concept that is essential in explaining what happens in collisions and explosions.

An important principle:

The total momentum of a system remains constant provided that no external forces act on the system.

This has important implications in the study of collisions. In simple terms, we can say that the **total momentum before = total momentum after**. The key thing is that share of the momentum may change.

The system may consist of several elements, each of which has its own momentum. The **total momentum** is the **sum** of all these different momenta. As long as the total momentum remains the same, momentum can be shared out differently between each element. In other words, each body can exchange momentum with the other bodies, and get a different share, as long as the total momentum stays the same.

If one element hogs most of the momentum, the others won't have much. We can model a closed system using the dodgems in a fairground (*Figure 134*).

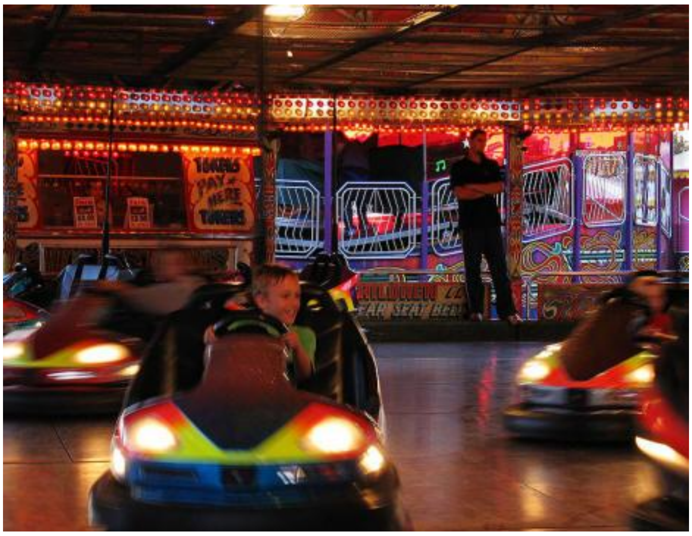


Figure 134 Dodgems at a fairground (Photo by Andrew Dunn. Wikimedia Commons)

Each car has the same mass, but we can imagine them having different velocities. Remember that velocity has a value (the speed) and a direction. In the diagram below, we can see that the velocities are all random, but the sum of all the momenta is zero (*Figure 135*).

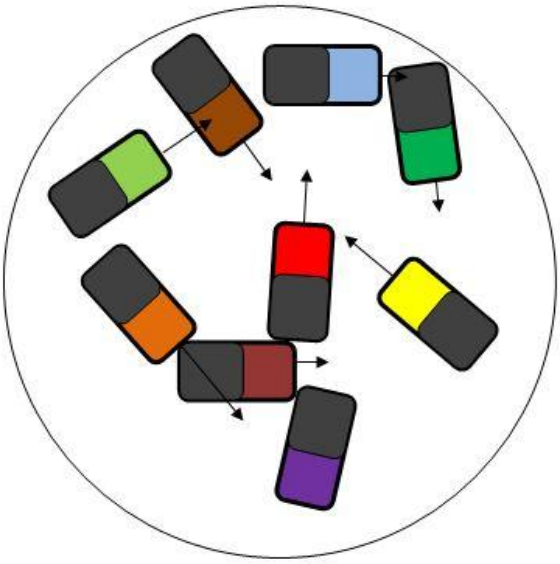


Figure 135 Velocities summing to zero

If it were not, the change in momentum would result in an overall force, resulting in movement (Newton II).

A few moments later, we might see (*Figure 136*).

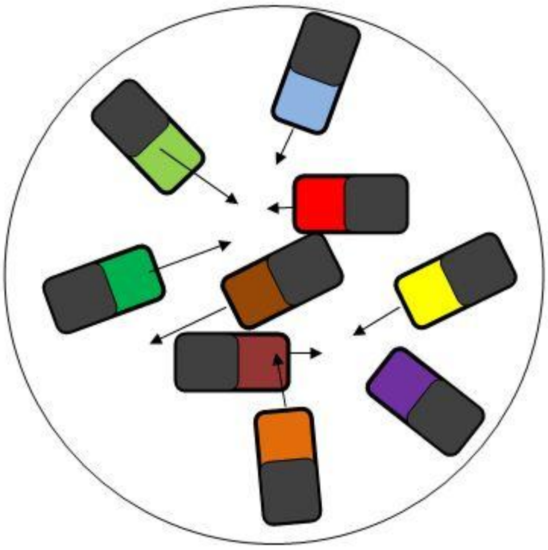


Figure 136 In dodgems, the velocities are changing all the time, but the overall velocity remains zero.

You can see that all the different cars have different velocities, hence different momenta. But the overall momentum remains the same. The difference in momentum is zero. This arises because forces act in pairs (Newton III).

We will only consider the momentum of two objects, acting in a straight line. This has important implications in the study of collisions. Remember:

The total momentum of a system remains constant provided that no external forces act on the system.

5.121 Collisions in 1 dimension

Momentum is always conserved in **collisions**.

If objects bounce off each other, the collision is **elastic**. If the total kinetic energy is the same (**conserved**) at the end as it is at the start, then the collision is **perfectly elastic**. The rebound of particles against each other tends to be **perfectly elastic**. A tennis ball bouncing off the floor is not perfectly elastic as it can lose up to 25 % of its kinetic energy in doing so.

If some kinetic energy is lost, converted into heat or light, then the collision is **inelastic**.

Think about two objects travelling in the same direction. The table below shows the properties of the objects:

Property	***Large Object***	***Small Object***
Mass	M	m
Initial velocity	u_1	u_2
Final velocity	v_1	v_2

We can show this as a diagram (*Figure 137*).

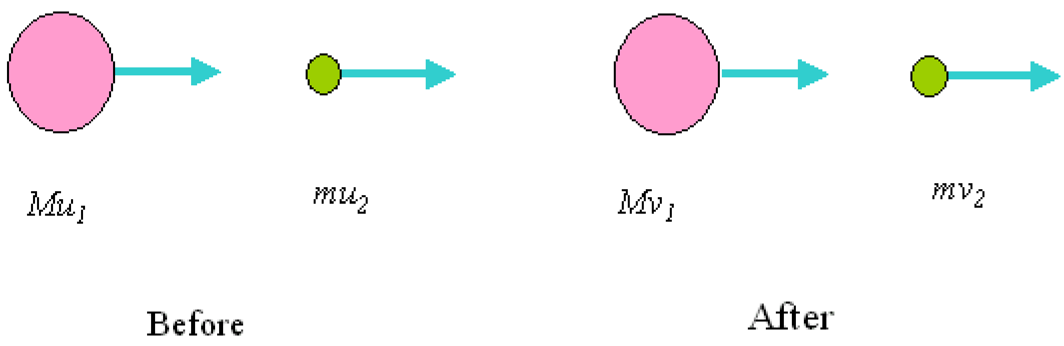


Figure 137 Momentum is conserved

There are two important principles here:

1. **Conservation of momentum**:

Total momentum before = total momentum after

$$Mu_1 + mu_2 = Mv_1 + mv_2 \text{ .... } \textit{Equation 102}$$

2. **Energy is conserved**

Total energy before = total energy after

$$\tfrac{1}{2}M(u_1)^2 + \tfrac{1}{2}m(u_2)^2 = \tfrac{1}{2}M(v_1)^2 + \tfrac{1}{2}m(v_2)^2 + E$$

...Equation 103

The term E is the energy that is **lost** in the collision. In a perfectly elastic collision $E = 0$.

When doing momentum calculations, always be careful about the directions you are using.

5.122 Momentum in Explosions

An explosion not just where something goes BANG. For all those who are disappointed with this, here is an explosion (*Figure 138*).

Figure 138 An explosion that goes bang!

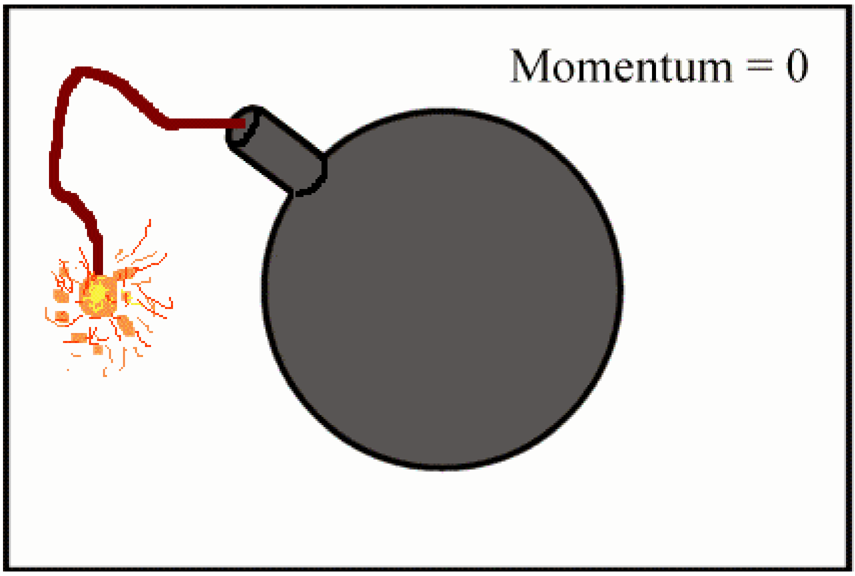


Figure 139 An animated gif of an explosion that goes bang.

Explosions in physics are much tamer affairs. The following are explosion systems:

- A rocket.
- Two physics trolleys pushed together on their spring plungers and then released.
- A gun firing a bullet.
- A student on a skateboard throwing a medicine ball (a heavy ball).
- A heavy nucleus ejecting an alpha particle.

In physics it is any situation where there is **zero momentum at the start**. Since linear momentum is conserved, it means that the total momentum at the end must be zero.

The general principles to work with are:

- Momentum is always conserved in explosions.
- Momentum before = momentum after = 0.
- There is zero velocity before.
- Then the two components move in opposite directions with equal momentum.
- The momenta add up to 0 because the directions are opposite.

Consider two railway wagons that are buffered up very tightly and the springs in the buffers are ready to push them apart (*Figure 140*). They are NOT coupled.

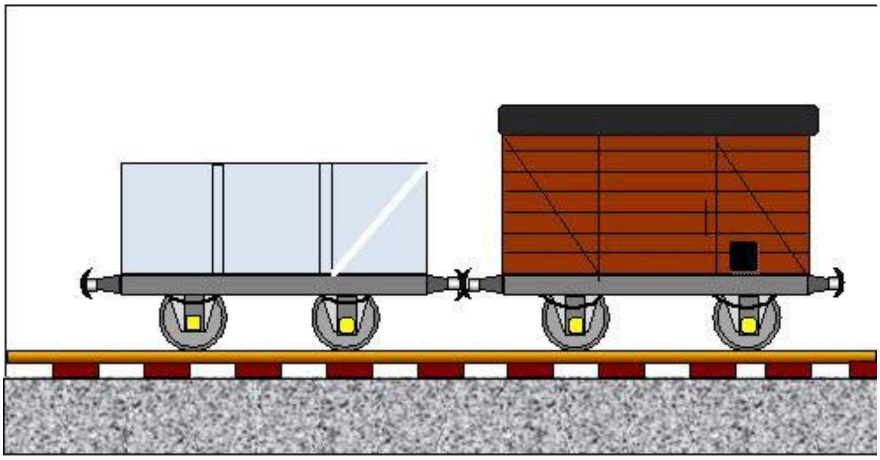


Figure 140 Two railway wagons about to fly apart

When the wagons are released, they fly apart in opposite directions as in the picture below (*Figure 141*).

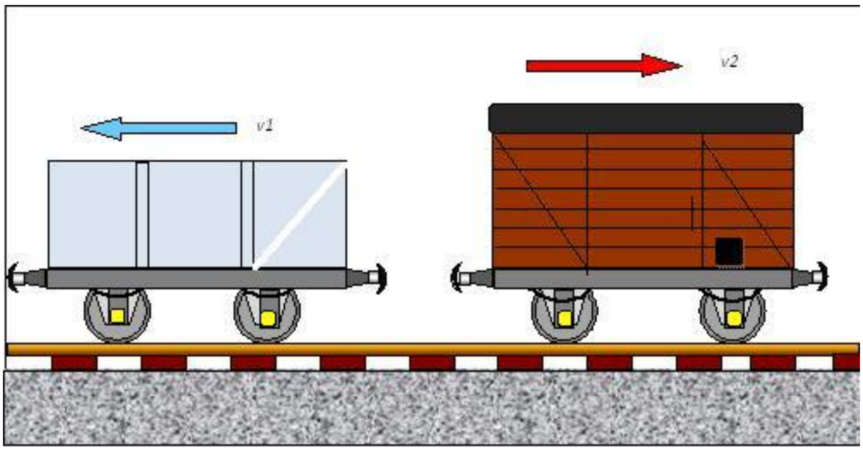


Figure 141 The wagons have now flown apart.

Since momentum is conserved and the momentum at the start was zero, we can write:

$$0 = m_1v_1 + m_2v_2$$ ………… *Equation 104*

which we can rearrange to:

$$m_1v_1 = -m_2v_2$$ ………… *Equation 105*

Worked example
Wagons 1 and wagon 2 are buffered up tightly together, but NOT coupled together. The brakes on the wagons are released at the same time. The release of the springs makes wagon 2 move to the right at a velocity of 0.10 m s^{-1}. What is the velocity of wagon 1?
Answer Momentum at start = 0: Momentum at end = 0 Momentum of wagon 2 = 20000 kg × +0.1 m s^{-1} = +2000 kg m s^{-1} Momentum of wagon 1 = 0 kg m s^{-1} - +2000 kg m s^{-1} = -2000 kg m s^{-1} Velocity of wagon 1 = -2000 kg m s^{-1} ÷ 15 000 kg = **-0.13 m s^{-1}** from right to left.

In the physics lab the situation can be recreated using physics trolleys (*Figure 142*).

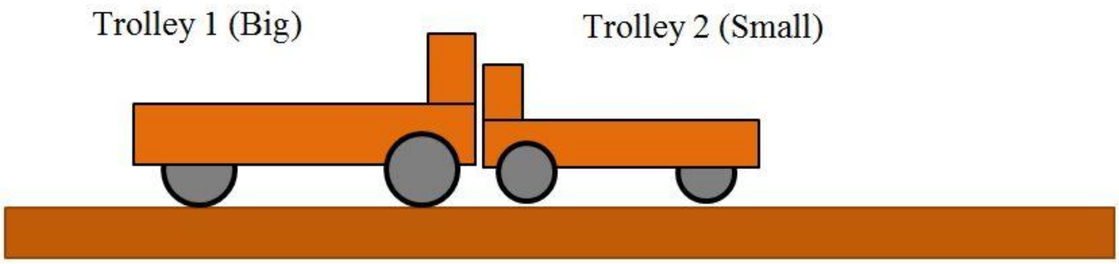


Figure 142 Two physics trolleys about to spring apart

The trolleys are pushed together and then spring apart (*Figure 143*).

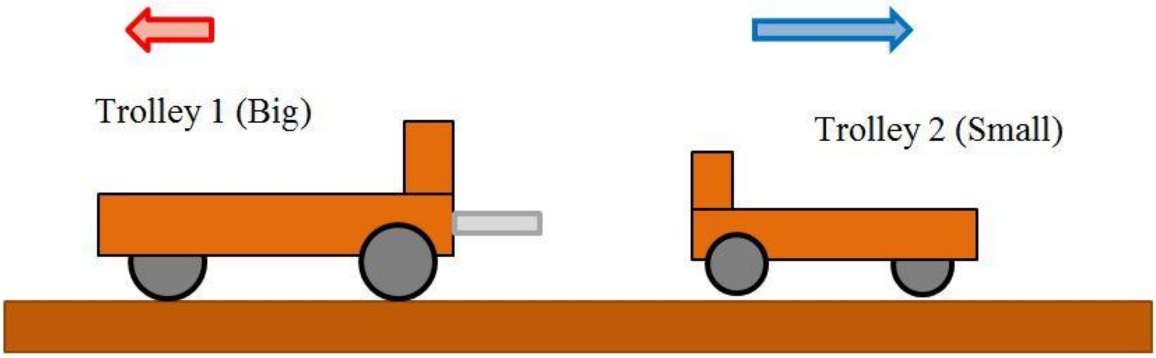


Figure 143 Two physic trolleys having sprung apart going in opposite directions.

Let the big trolley have mass M and the small trolley have mass m. At the start, the momentum of both trolleys is 0.

We can write down the momenta at the end:

Momentum of the big trolley = MV
Momentum of the small trolley = mv

We know that the momentum at the start will be 0. Therefore, the momentum at the end will be zero due to the conservation of momentum. So, we can now write:

$$MV + mv = 0$$ ……………. *Equation 106*

We rearrange this to:

$$-MV = mv$$ …………. *Equation 107*

The minus sign is there to tell us that the big trolley is going from **right to left**.

Worked Example
The big trolley has a mass of 1.2 kg, and the small trolley has a mass of 0.80 kg. The velocity of the small trolley is measured at 2.0 m s^{-1} from left to right. Calculate the velocity of the big trolley.
Answer
0.8 kg × +2.0 m s^{-1} = 1.2 kg × -V -V = 1.2 kg ÷ 1.6 kg m s^{-1} = **-0.75 m s^{-1}** (i.e. 0.75 m s^{-1} from right to left)

In these cases, the energy for the explosion was provided by springs which were part of the vehicle. The mass of each vehicle was the same after as before.

We can derive a single expression for the recoil velocity. Consider an unstable nucleus of mass M. It emits an alpha particle of mass m. Immediately after the explosion event, the daughter nucleus has a mass N where:

$$N = M - m \quad \text{.............. Equation 108}$$

This is shown below (*Figure 144*).

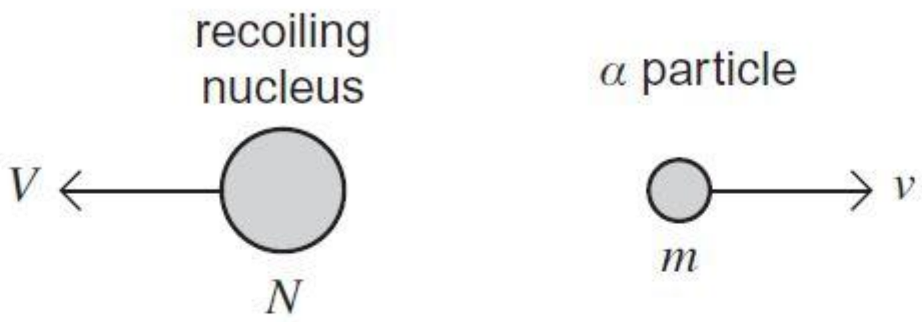


Figure 144 Recoil of a daughter nucleus after the emission of an alpha particle

The nucleus has a velocity of V from right to left, while the alpha particle has a velocity v from left to right.

We can derive an expression for the velocity V for the nucleus. This is an explosion, so the initial momentum $= 0$. By the conservation of momentum:

$$0 = NV + mV \quad \text{............ Equation 109}$$

$$mv = -NV \text{ .... } \textit{Equation 110}$$

Therefore:

$$V = -\frac{mv}{N}$$

...............*Equation 111*

We can write this as:

$$V = -\frac{mv}{M - m}$$

.... *Equation 112*

Note that in this case, N is the mass of the nucleus **after** it has ejected the alpha particle. If it had a mass M before the explosion, we would have to take the loss of mass into account:

$$N = M - m$$

Therefore, we have to be careful to make sure that we read the question carefully - RTQ, *Read The Question*.

(I can't talk - I have often not read the question. When my physics teacher got mad at my homework once, he wrote RTFQ. I was assured by my housemaster that it meant *Read the <u>Full</u> Question*...)

In the case above, the unstable nucleus lost mass. In the next question, the energy for the explosion comes from a charge in the behind the shell. When a reaction happens, there is negligible loss of mass. The mass of the reactants is the same as the mass of the products.

5.123 Energy in Collisions

Energy is always **conserved** as it must be in any physics system. All moving objects have **kinetic** energy, which we know from GCSE has the relationship:

$$E_k = \frac{1}{2}mv^2$$

.... *Equation 113*

Total energy before = total energy after.

While all the energy of the moving objects before the collision is assumed to be kinetic energy, the energy after the collision may NOT be all kinetic energy. If we do the momentum calculation and work out the velocity of each object, we will find that the **momentum is conserved**, but the kinetic energy before is greater than the kinetic energy after. Some kinetic energy is **lost**. It ends up as **noise** and **internal energy**. This is shown in the equation:

$$\tfrac{1}{2}M(u_1)^2 + \tfrac{1}{2}m(u_2)^2 = \tfrac{1}{2}M(v_1)^2 + \tfrac{1}{2}m(v_2)^2 + E$$

...*Equation 114*

The term E is the difference between the kinetic energy before and the kinetic energy after. When $E = 0$, we have a **perfectly elastic collision**.

Since the velocity is squared when calculating the kinetic energy, the kinetic energy is always **positive**, regardless of the direction of the movement. Energy is a scalar.

Most collisions are **inelastic**, which means that kinetic energy is transferred to other sorts of energy.

Kinetic energy after cannot be greater than before, unless external work is done on the system.

5.124 Kinetic energy in elastic collisions

Perfectly elastic collisions only happen in particle physics. They can be modelled with a **linear air track** in which the vehicles carry mutually repelling magnets (*Figure 145*)

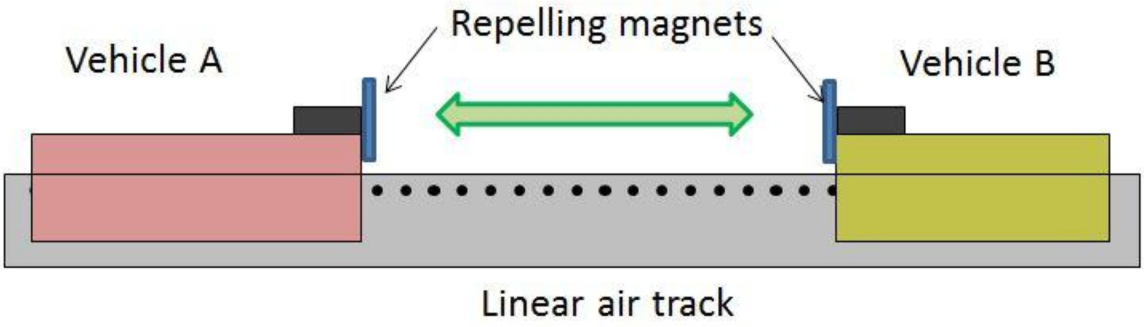


Figure 145 Modelling a perfectly elastic collision

Newton's cradle is a toy that also models conservation of momentum and conservation of kinetic energy (*Figure 146*).

Figure 146 Newton's Cradle

Drop one ball and one ball will swing up at the other end (*Figure 147*).

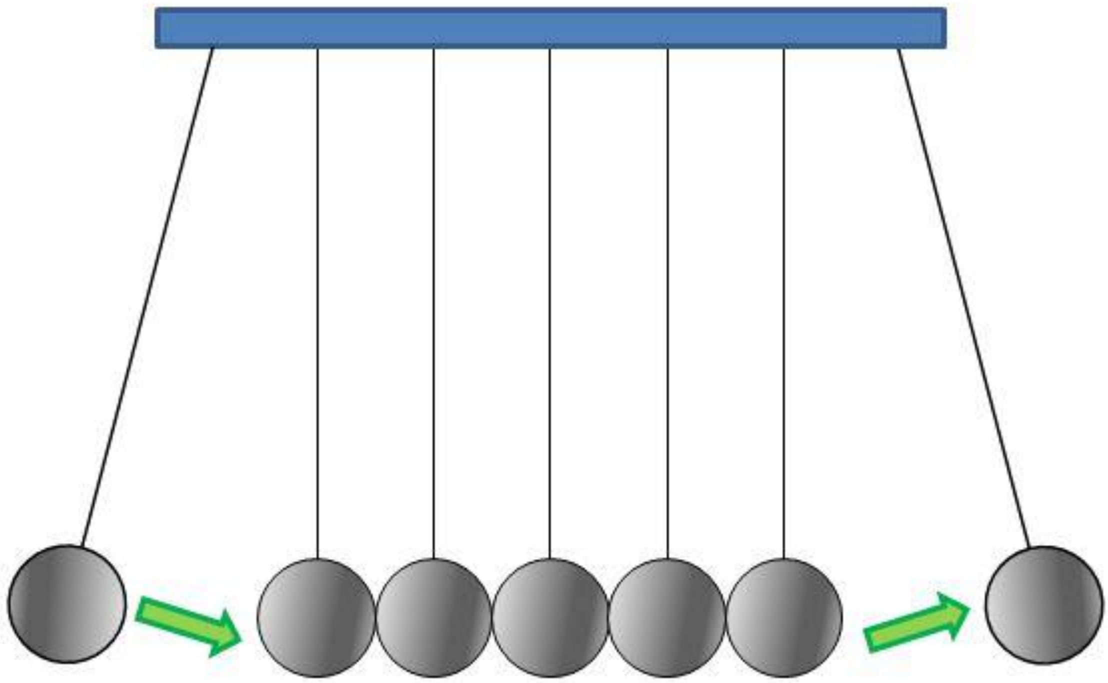


Figure 147 Newton' Cradle in action

We would never get two balls swinging up. Consider the rules:

- Momentum has to be conserved.
- Kinetic energy is conserved.

Let the velocity of the ball be v, and the mass be m. If one ball comes up when two balls are dropped, to conserve momentum we need:

$$2mv = m \times 2v$$

.... *Equation 115*

The velocity at the end is half what it was at the start. There is no reason why this couldn't happen but look at the kinetic energy. Kinetic energy at the start is:

$$E_{\mathrm{k}} = \frac{1}{2} \times 2mv^2$$

.... *Equation 116*

Kinetic energy at the end is:

$$E_k = \frac{1}{2} \times m(2v)^2 = 2mv^2 \neq mv^2$$

.... *Equation 117*

This means that the kinetic energy is doubled. To achieve this, the speed is has to be 1.4 (√2) times what it was at the start. This is not consistent with the momentum. Therefore, it cannot happen.

In reality, the collisions are not perfectly elastic. Some of the kinetic energy is dissipated as **sound** (you hear the balls clicking as they collide).

5.125 Kinetic energy in Explosions (A level only)

Earlier on in the tutorial, we considered the case of an unstable nucleus emitting an alpha particle. The unstable nucleus has a mass M. It emits an alpha particle of mass m. Immediately after the explosion event, the daughter nucleus has a mass N where:

$$N = M - m$$ .... *Equation 118*

This is shown below (*Figure 148*).

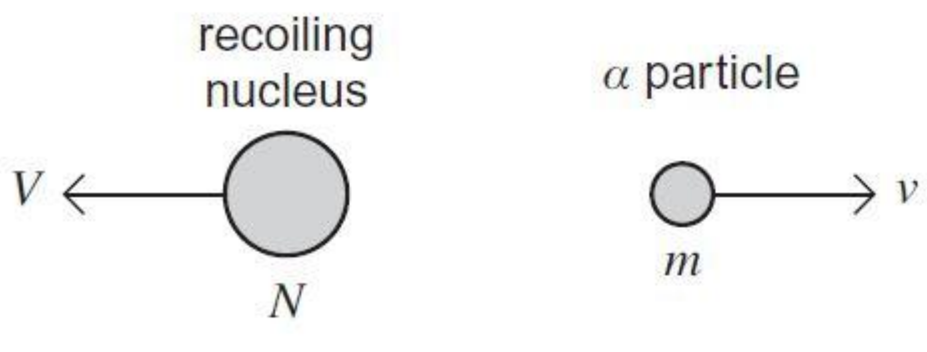


Figure 148 An unstable nucleus emits an alpha particle

The nucleus has a velocity of V from right to left, while the alpha particle has a velocity v from left to right. Now let's think about the kinetic energy. The kinetic energy is a scalar, so the directions of the velocities are not important. Let's assume that the unstable nucleus is stationary, so that the original kinetic energy is zero.

We are going to derive an expression that shows how the kinetic energy of the emitted particle is linked with the total kinetic energy. If we know the energy of the decay, we can use it to predict the kinetic energy of the alpha particle (hence its speed). Measuring the speed of an alpha particle by direct measurement is not easy.

So, we can write:

Total energy = kinetic energy of the recoiling nucleus + kinetic energy of the alpha particle

$$E_{\mathrm{k\,tot}} = \frac{1}{2}NV^2 + \frac{1}{2}mv^2$$

.... *Equation 119*

We have an expression for V derived from the momenta of the two particles:

$$V = -\frac{mv}{N}$$

.... *Equation 120*

So, we can substitute:

$$E_{\mathrm{k\,tot}} = \frac{1}{2}N\left(\frac{mv}{N}\right)^2 + \frac{1}{2}mv^2$$

.... *Equation 121*

This becomes:

$$E_{\mathrm{k\,tot}} = \frac{1}{2}\left(\frac{m}{N}\right)mv^2 + \frac{1}{2}mv^2$$

.... *Equation 122*

We can look at the kinetic energy of the alpha particle, which we will call E_α which is given by:

$$E_\alpha = \frac{1}{2}mv^2$$

.... *Equation 123*

So, we can now write:

$$E_{\text{k tot}} = E_\alpha \left[\left(\frac{m}{N}\right) + 1\right]$$

.... *Equation 124*

Therefore, rearranging *Equation 124* to make E_α the subject:

$$E_\alpha = \frac{E_{\text{k tot}}}{\left(\frac{m+N}{N}\right)}$$

.... *Equation 125*

Equation 125 rearranges to:

$$E_\alpha = \frac{E_{\text{k tot}}N}{m+N} = \left(\frac{N}{m+N}\right)E_{\text{k tot}}$$

.... *Equation 126*

I have included this seemingly rather contrived derivation as it formed part of a question in an A2 paper. (Open confession: It stumped me, which irritated me intensely. It also stumped my colleague.)

At university you will study the **Q-value**, which is the same as kinetic energy of the decay, $E_{\text{k tot}}$ here. It is the difference of the rest energy of the parent nucleus, E_{P}**,** and the combined rest energies of the daughter nucleus, E_{D}, and the emitted particle, E_α.

$$Q = (E_{\text{D}} + E_\alpha) - E_P$$

………… *Equation 127*

The energies are the binding energies of the nuclei.

We can use the integer mass numbers for the masses m and N. For the alpha particle, the mass number is 4.

5.126 Momentum in Two Dimensions (OCR Syllabus)

This is a requirement of the OCR, Cambridge International, and Cambridge Pre-U syllabuses. It is not on other syllabuses.

Consider a ball bearing of mass m_1 kg travelling at a velocity u_1 m s^{-1} at an angle of θ_1 to the horizontal. Its total momentum is p_1 N s. See *Figure 149*.

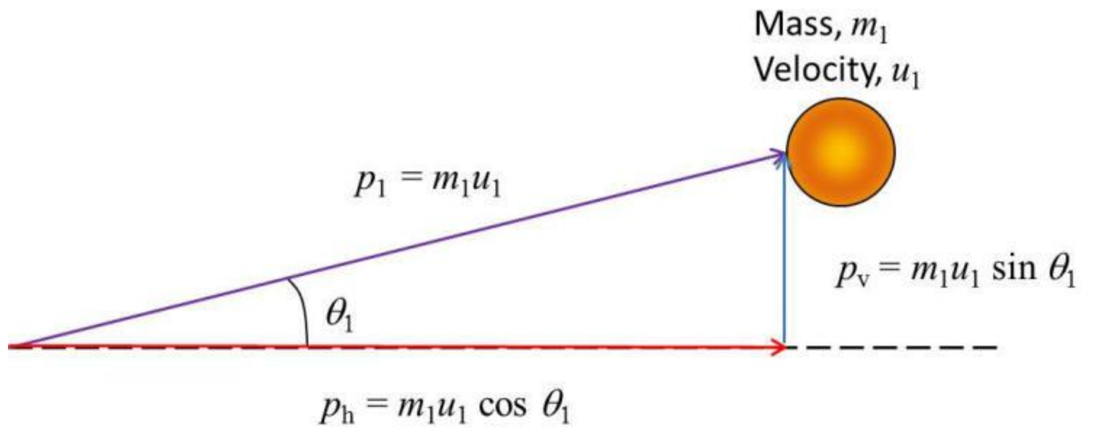


Figure 149 Momentum at an angle

Since momentum is a vector, we can resolve it into vertical and horizontal components, as shown on the picture. Now consider a second ball bearing of a different mass m_2 kg travelling at a different velocity u_2 m s^{-1} at a different angle of θ_2 to the horizontal. Its total momentum is p_2 N s (*Figure 150*).

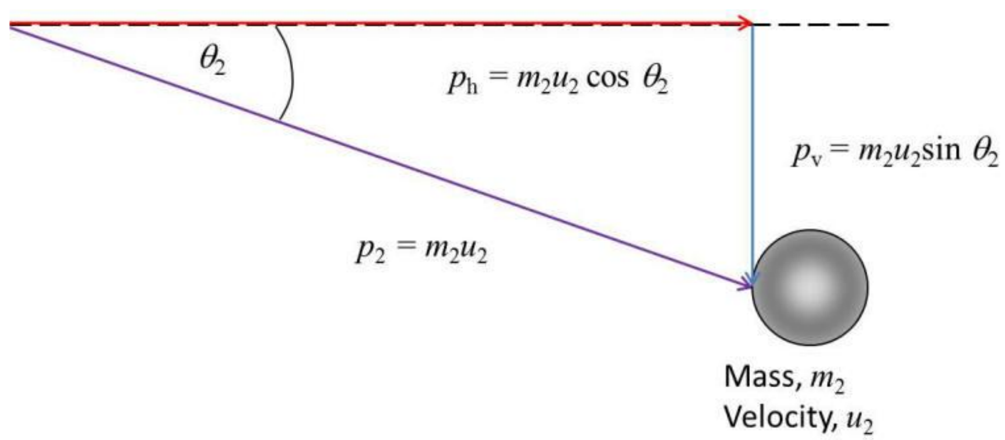


Figure 150 Momentum at an angle

Let's make the two ball bearings collide (*Figure 151*).

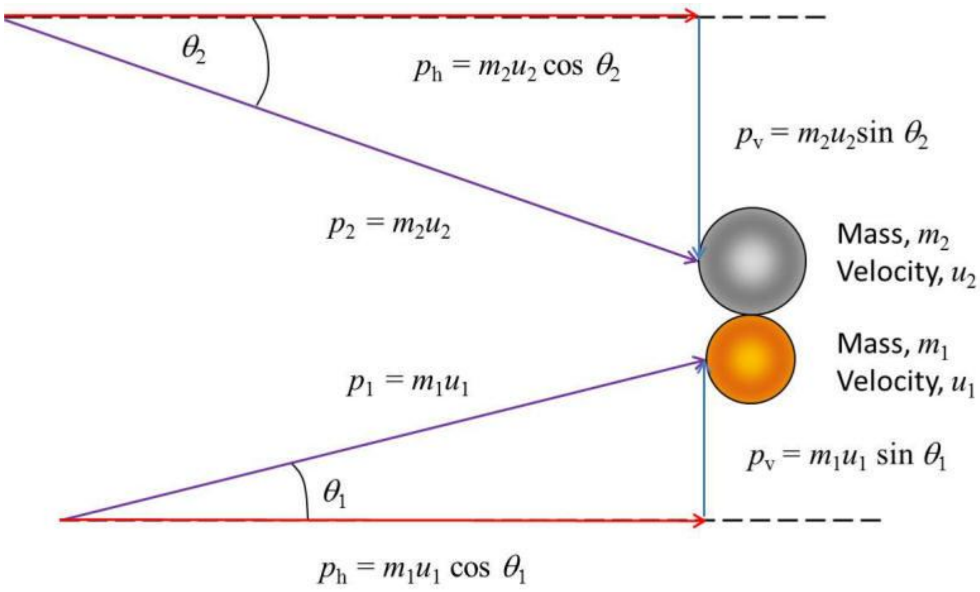


Figure 151 Collision between two ball bearings at an angle

After the collision we see *Figure 152*.

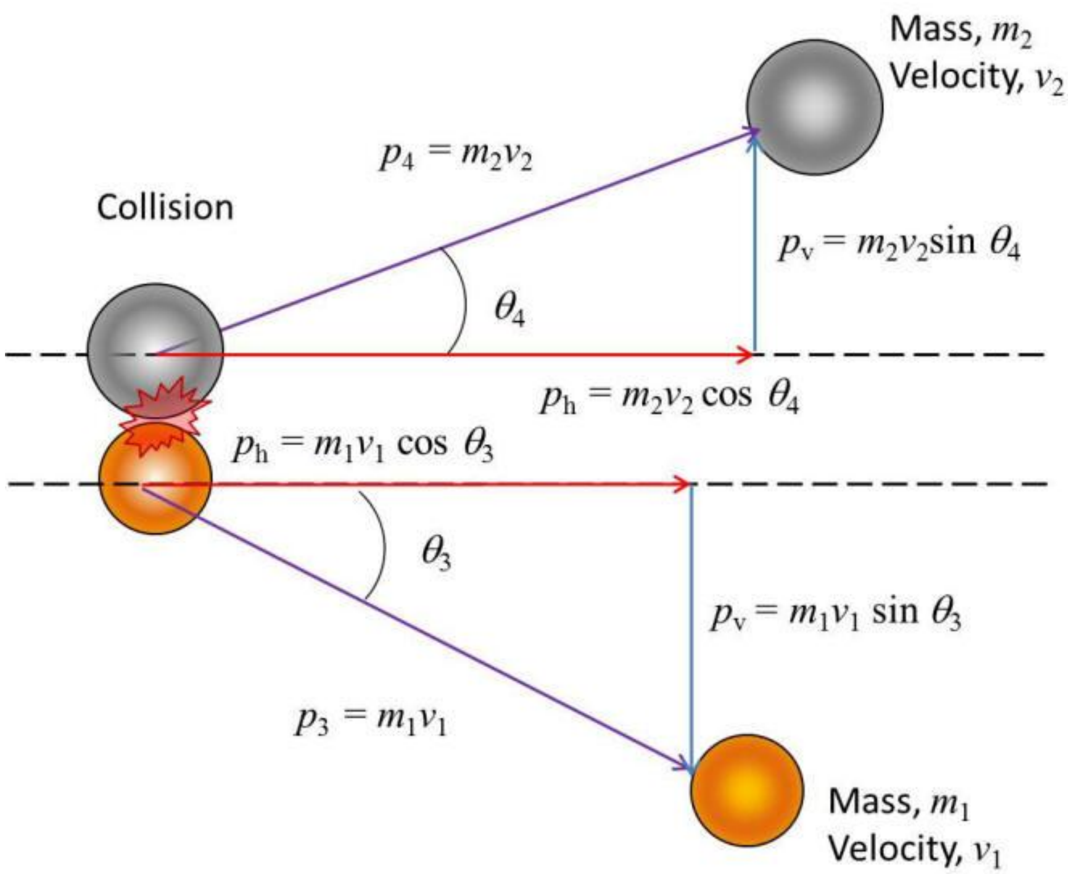


Figure 152 After the collision between two ball bearings at an angle

We will model the two ball bearings as **point masses**.

It all looks rather complicated, but if we follow through logically, we will find a way through. We need to look at some important rules in handling problems like this:

- Momentum is **conserved** in both **horizontal** and **vertical** directions.
- We must keep the horizontal components **separate** from the vertical components.
- We need to keep to the convention that **left to right** is **positive**, and that going **upwards** is **positive**.

Before the collision:

Total horizontal momentum =

$$p_1 \cos \theta_1 + p_2 \cos \theta_2 = m_1 u_1 \cos \theta_1 + m_2 u_2 \cos \theta_2 \text{ …………. } \textit{Equation 128}$$

Total vertical momentum =

$$p_1 \sin \theta_1 + p_2 \sin \theta_2 = m_1 u_1 \sin \theta_1 + m_2 u_2 \sin \theta_2 \text{ ……… } \textit{Equation 129}$$

After the collision:

Total horizontal momentum =

$$p_3 \cos \theta_3 + p_4 \cos \theta_4 = m_1 v_1 \cos \theta_3 + m_2 v_2 \cos \theta_4 \text{ ………… } \textit{Equation 130}$$

Total vertical momentum =

$$p_3 \sin \theta_3 + p_4 \sin \theta_2 = m_1 v_1 \sin \theta_3 + m_2 v_2 \cos \theta_4 \text{ ………… } \textit{Equation 131}$$

The worked example below shows how these equations are used with numbers in.

<u>*Worked example*</u>

Let's make m_1 = 1.5 kg, u_1 = 2.0 m s^{-1}, and θ_1 = 25°.

Let's make m_2 = 2.0 kg, u_2 = 1.8 m s^{-1}, and θ_2 = 34°. The two objects collide.

After the collision, mass m_1 has a velocity of 1.5 m s^{-1} and an angle of 40° and mass m_2 has a velocity of v m s^{-1} at an angle of 20°. Work out the velocity of mass m_2.

Answer

Work out the momentum vectors before:

Momentum of mass m_1 = 1.5 kg × 2.0 m s^{-1} = **3.0 N s**

Momentum of mass m_2 = 2.0 kg × 1.8 m s^{-1} = **3.6 N s**

Now work out the horizontal and vertical momenta by summing the horizontal and vertical components of the momentum vectors:

Total horizontal momentum = (3.0 N s × $\cos$ 25) + (3.6 N s × $\cos$ 34)

= 2.719 N s + 2.985 N s = **5.704 N s** (Both are from left to right so are positive.)

Total vertical momentum = (3.0 N s × $\sin$ 25) + -(3.6 N s × $\sin$ 34)

= 1.268 N s + -2.013 N s = **-0.745 N s** (Remember the minus sign as m_2 is moving downwards.)

We will round to an appropriate number of significant figures later.

Work out the momentum vectors after:

Momentum of mass m_1 = 1.5 kg × 1.5 m s^{-1} = **2.25 N s**

Momentum of mass m_2 = 2.0 kg × v m s^{-1} = **2.0 v N s**

Now work out the horizontal and vertical momenta by **summing** the horizontal and vertical components of the momentum vectors:

Total horizontal momentum = (2.25 N s × $\cos$ 40) + (2.0 v N s × $\cos$ 20)

= 1.930 N s + 1.879 v N s = **5.704** N s (Both are from left to right so are positive.)

Therefore, we can work out the horizontal component:

$$1.879\ v = 5.704\ \text{N s} - 1.930\ \text{N s} = \mathbf{3.774\ N\ s}$$

$$v = \mathbf{2.009}\ \text{m s}^{-1}.$$

This gives us the **horizontal** component.

Now we can work out the **vertical** component:

Total vertical momentum = $-(2.25\ \text{N s} \times \sin 40) + (2.0\ v\ \text{N s} \times \sin 20)$

$$= -1.446\ \text{N s} + 0.6840\ v\ \text{N s} = -0.745\ \text{N s}$$

(Remember the minus sign as m_1 is moving downwards.)

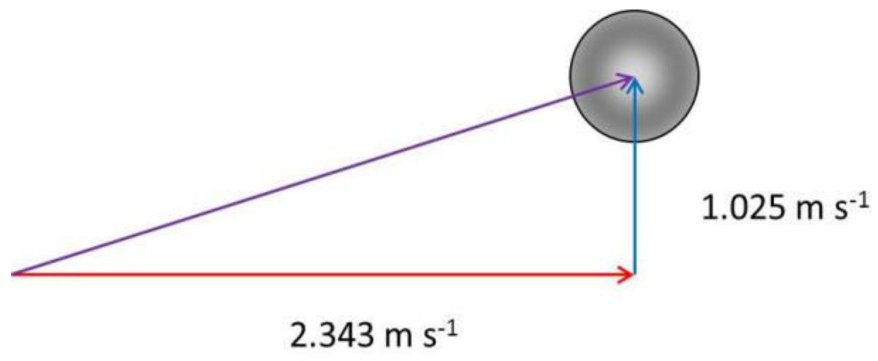
Therefore:

$$0.6840\ v = -0.745\ \text{N s} - -1.4460\ \text{N s} = \mathbf{0.7010\ N\ s}$$

$$v = \mathbf{1.025\ m\ s^{-1}}$$

This gives us the **vertical** component.

So, the resultant velocity is summarised here:



So, the resultant velocity is the vector sum:

$$v_{res}^2 = (2.343\ \text{m s}^{-1})^2 + (1.025\ \text{m s}^{-1})^2 = 6.5402\ \text{m}^2\ \text{s}^{-2}$$

$$v = 2.557\ \text{m s}^{-1} = \underline{\mathbf{2.6\ m\ s^{-1}}}\ \text{(to 2 s.f.)}$$

The calculation is quite involved. The main difficulty here is making sure that the signs are right.

5.127 Rocket Science

A rocket moves because fuel is burned to gases which move with a certain velocity. Mass is conserved in the reaction, so that if 10 kg of fuel is burned, 10 kg of gases are produced. As the gases move with a velocity, there has to be momentum. As momentum is conserved there has to be an equal and opposite momentum applied to the rocket.

If we bring time in, we can convert the momentum to a force. So, if 10 kg s^{-1} of fuel is burned, then 10 kg s^{-1} of gas is produced. If we know the velocity, we can work out the change in momentum per second, which is force (Newton II).

Forces act in pairs (Newton III) so there is an equal and opposite force on the rocket. (*Figure 153*)

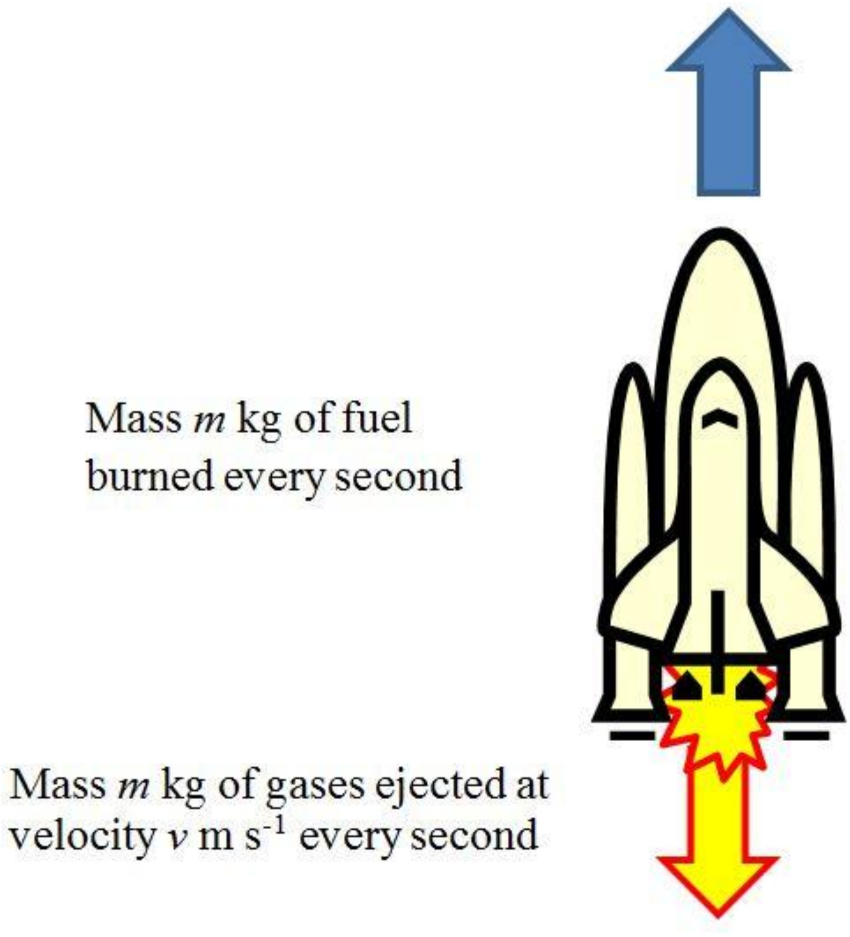


Figure 153 Forces on a rocket

Remember that the upwards thrust F_E from the engines is opposed by the weight W of the rocket, which acts vertically downwards. Therefore, the resultant force F_R is given by:

$$F_R = F_E - W$$ .... *Equation 132*

Always take into account the weight of the rocket. Failure to do so will give you the wrong answer. You will get the acceleration that would be achieved in outer space.

Change in momentum of mv kg m s^{-1} every second. Since $\Delta p = F\Delta t$, this results in a force. Newton's Laws apply. Forces act in pairs (Newton III). Force results in acceleration (Newton II).

5.128 Modelling the Motion of the Rocket (Extension)

Using a spreadsheet, we can **model** the acceleration of the rocket. We use the following equations:

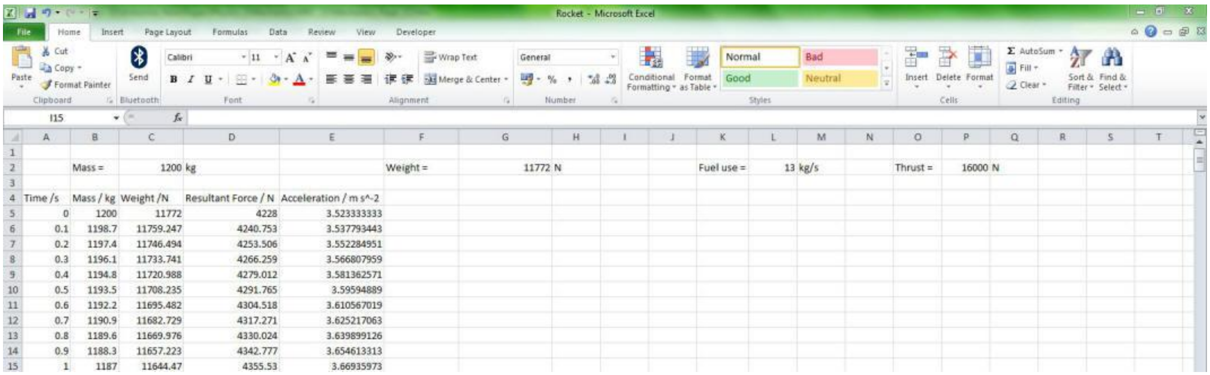
Mass of rocket (kg) = Original mass (kg) - [rate of fuel use (kg s^{-1}) × time (s)]

Weight (N) = Mass of Rocket (kg) × 9.81 N kg^{-1}

Resultant force (N) = Thrust (N) - Weight (N)

Acceleration (m s^{-2}) = Resultant force (N) ÷ Mass (kg)

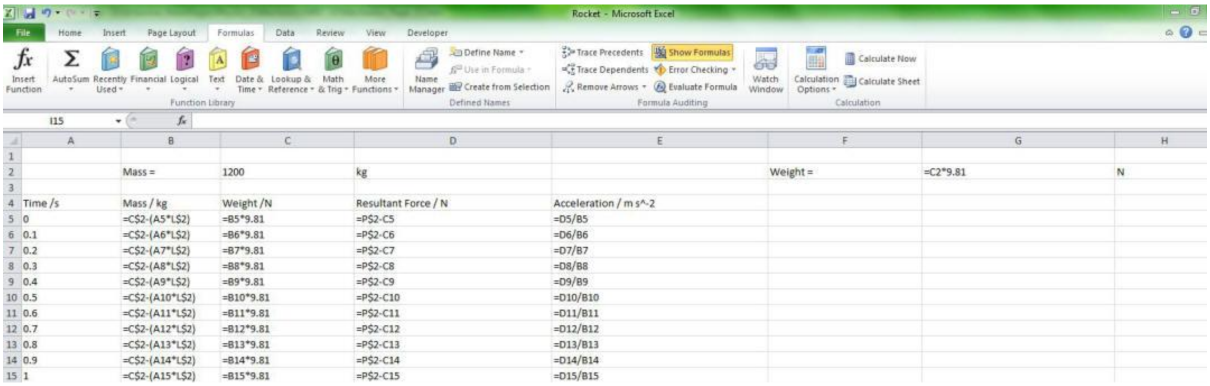
If you are a dab hand at spreadsheets, you can try it for yourself. In generating the spreadsheet, I used the data from Question 5.12.9. I have used a sampling time of 0.1 s. Here are the data for the first second (*Figure 154*).



	A	B	C	D	E	F	G	H	K	L	M	O	P	Q
1														
2		Mass =	1200	kg		Weight =	11772	N	Fuel use =	13	kg/s	Thrust =	16000	N
3														
4	Time /s	Mass / kg	Weight /N	Resultant Force / N	Acceleration / m s^-2									
5	0	1200	11772	4228	3.523333333									
6	0.1	1198.7	11759.247	4240.753	3.537793443									
7	0.2	1197.4	11746.494	4253.506	3.552284951									
8	0.3	1196.1	11733.741	4266.259	3.566807959									
9	0.4	1194.8	11720.988	4279.012	3.581362571									
10	0.5	1193.5	11708.235	4291.765	3.59594889									
11	0.6	1192.2	11695.482	4304.518	3.610567019									
12	0.7	1190.9	11682.729	4317.271	3.625217063									
13	0.8	1189.6	11669.976	4330.024	3.639899126									
14	0.9	1188.3	11657.223	4342.777	3.654613313									
15	1	1187	11644.47	4355.53	3.66935973									

Figure 154 Data for the first second after launch

And here are the formulae (*Figure 155*).



	A	B	C	D	E	F	G	H
1								
2		Mass =	1200	kg		Weight =	=C2*9.81	N
3								
4	Time /s	Mass / kg	Weight /N	Resultant Force / N	Acceleration / m s^-2			
5	0	=C$2-(A5*L$2)	=B5*9.81	=P$2-C5	=D5/B5			
6	0.1	=C$2-(A6*L$2)	=B6*9.81	=P$2-C6	=D6/B6			
7	0.2	=C$2-(A7*L$2)	=B7*9.81	=P$2-C7	=D7/B7			
8	0.3	=C$2-(A8*L$2)	=B8*9.81	=P$2-C8	=D8/B8			
9	0.4	=C$2-(A9*L$2)	=B9*9.81	=P$2-C9	=D9/B9			
10	0.5	=C$2-(A10*L$2)	=B10*9.81	=P$2-C10	=D10/B10			
11	0.6	=C$2-(A11*L$2)	=B11*9.81	=P$2-C11	=D11/B11			
12	0.7	=C$2-(A12*L$2)	=B12*9.81	=P$2-C12	=D12/B12			
13	0.8	=C$2-(A13*L$2)	=B13*9.81	=P$2-C13	=D13/B13			
14	0.9	=C$2-(A14*L$2)	=B14*9.81	=P$2-C14	=D14/B14			
15	1	=C$2-(A15*L$2)	=B15*9.81	=P$2-C15	=D15/B15			

Figure 155 and formulae for the spreadsheet

Here is the acceleration-time graph produced by the spreadsheet (*Figure 156*).

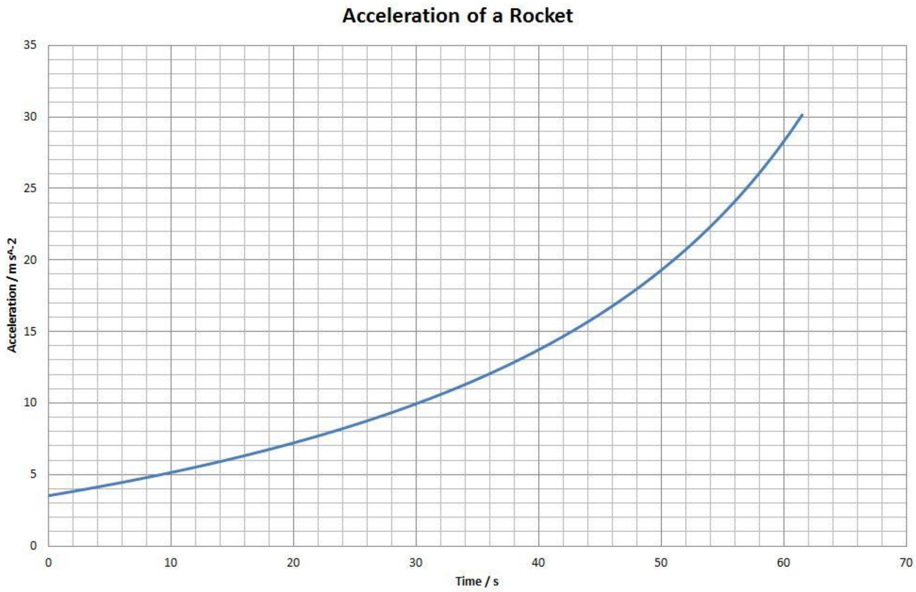


Figure 156 Acceleration-time graph using spreadsheet generated data.

You can see that the acceleration is not constant. It increases considerably as the fuel is used up:

- The mass is decreasing as fuel is used up.
- The weight is decreasing as the fuel is used up.
- The resultant force (= thrust - weight) is increasing.

The model is limited in that it does not take into account the air resistance, which will come into effect very quickly to act in the same direction as the weight. Also, it works by using a 0.1 s sampling method. This gives and approximation that is not so far out. The true answer would be obtained if we knew the function that links the acceleration of the rocket with time.

Extension (Challenge)

Try this out for yourself - Use the spreadsheet to generate data for a speed-time and a distance time graph. We know that the area under the acceleration-time graph gives the speed. In calculus notation:

$$v = \int_0^\infty a\,dt$$

.... *Equation 133*

If a is constant, the solution is:

$$v = u + at$$

.... *Equation 134*

However, in this case, the acceleration is not constant, as we have seen above. This is because the mass is decreasing as the fuel in the rocket is used up. As a result, the weight is decreasing. Therefore, the resultant force is increasing.

The speed can be worked out using:

$$v = u + at$$

.... *Equation 135*

The t term is the time **interval**, not the time value. If you use the time value, your speed values will be wrong. This relationship works because it is a formula that sums the previous speed value (*Figure 157*).

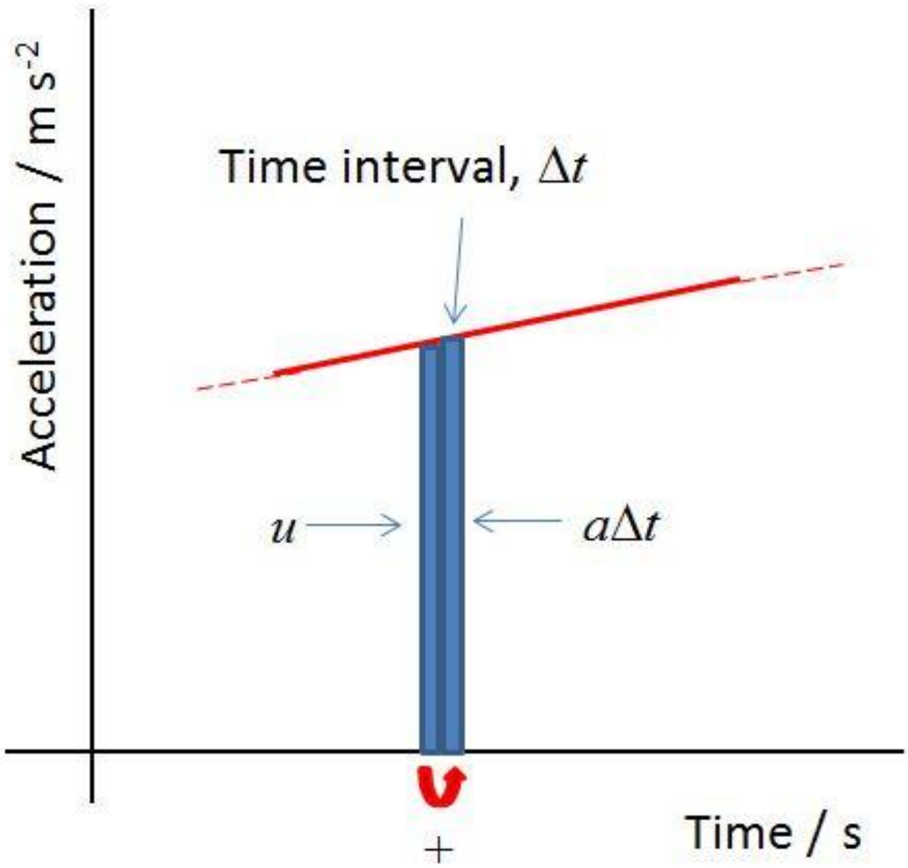


Figure 157 Using time intervals rather than value of time

To work out the speed time graph below, I have used a smaller time interval sample (0.01 s rather than 0.1 s as above). There is not much difference between the highest speeds obtained. With the 0.1 sample time, the speed at 61.5 s = 746 m s^{-1} compared with 747 m s^{-1} at 61.50 s with the 0.01 s sample time. Not much difference.

The speed time graph looks like this: (*Figure 158*).

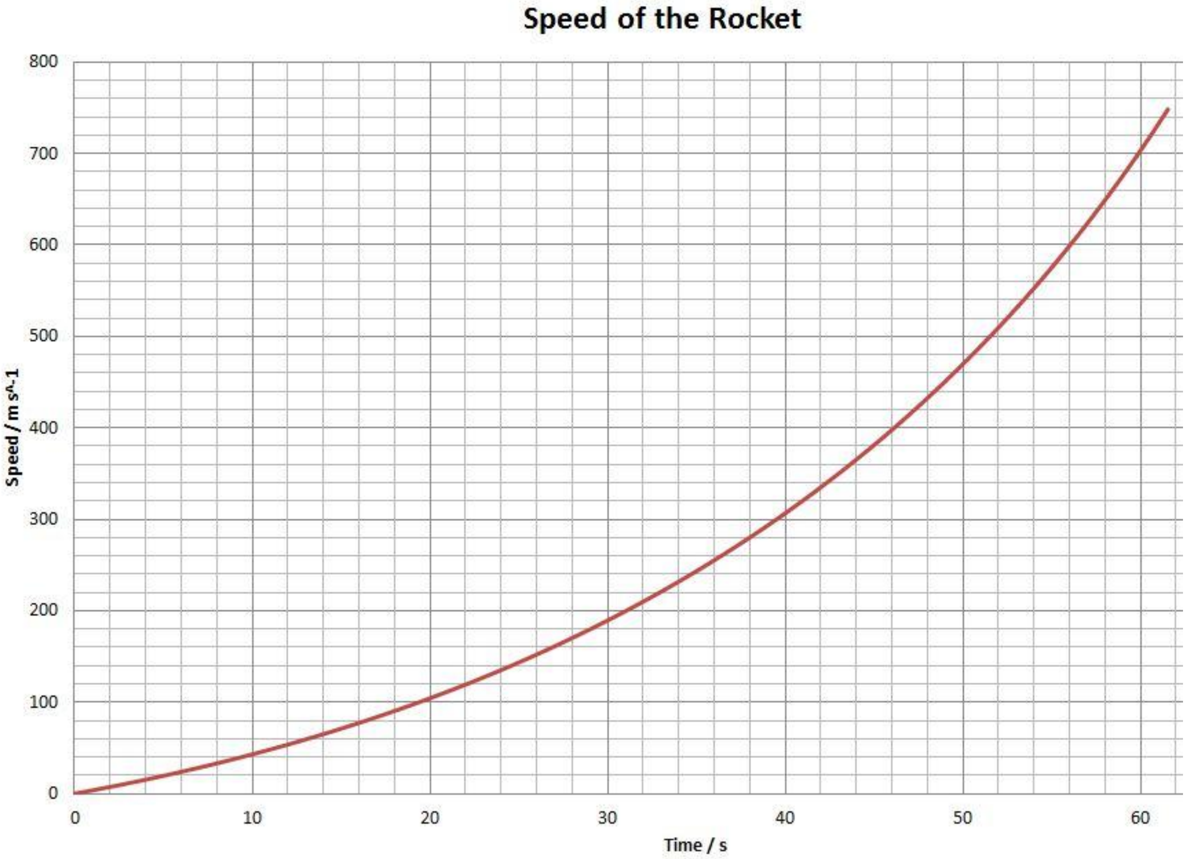


Figure 158 Speed time graph for a rocket using generated data

The distance would be worked out by counting the squares. This is tedious, especially if you lose your place. Here is the same graph showing the area under the graph being used to work out the distance. The area has been broken up into rectangles and triangles (*Figure 159*).

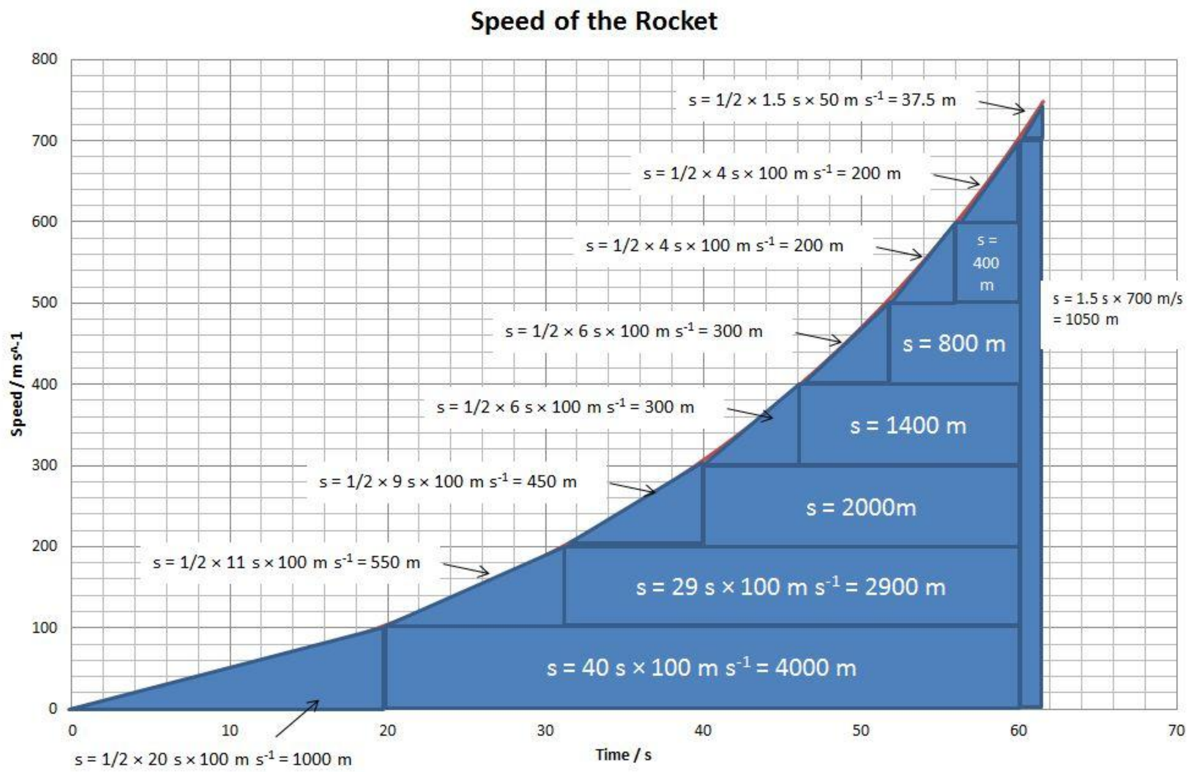


Figure 159 Working distance from the speed-time graph

If we add all the small areas together, we get:

s = 4000 m + 2900 m + 2000 m + 1400 m + 800 m + 400 m + 1050 m + 37.5 m + 200 m + 200 m + 300 m + 300 m + 450 m + 550 m + 1000 m = **15640 m** = 15600 m (3 s.f.)

A more precise answer can be gained by breaking up the graph into little strips of time interval 0.1 s or 0.01 s and working out the **area of each strip** using:

$$s = \frac{(u+v)t}{2}$$

.... *Equation 136*

or

$$s = ut + \frac{1}{2}at^2$$

.... *Equation 137*

Either of these equations gives you the **area of each little strip**, i.e. the distance travelled in 0.1 s or 0.01 s, depending on your chosen time interval. It does not give you the sum total of the strips so far. The maximum value (according to the data that I used) was about 7.5 m. You need to make a little summing formula in the next spreadsheet column.

Then you have add up all the areas of the little strips together:

- For the 0.1 s interval, the distance = 15504 m
- For the 0.01 s interval, the distance = 15528 m.

The distance time graph shown here is generated using the 0.01 s time interval (*Figure 160*).

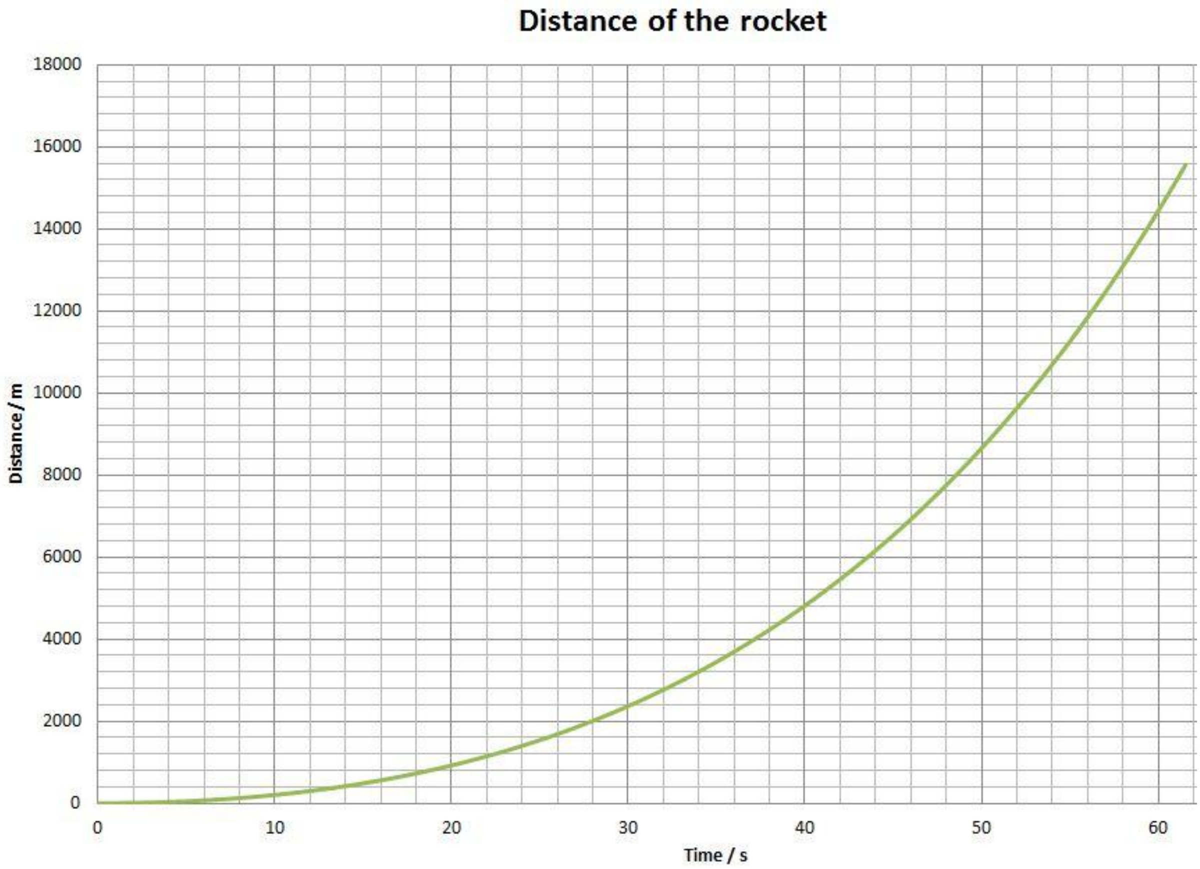


Figure 160 Distance-time graph from generated data.

The most precise value would obtained by using **calculus integration**. This would depend on knowing the function by which the acceleration was related to the time.

Tutorial 5.12 Questions

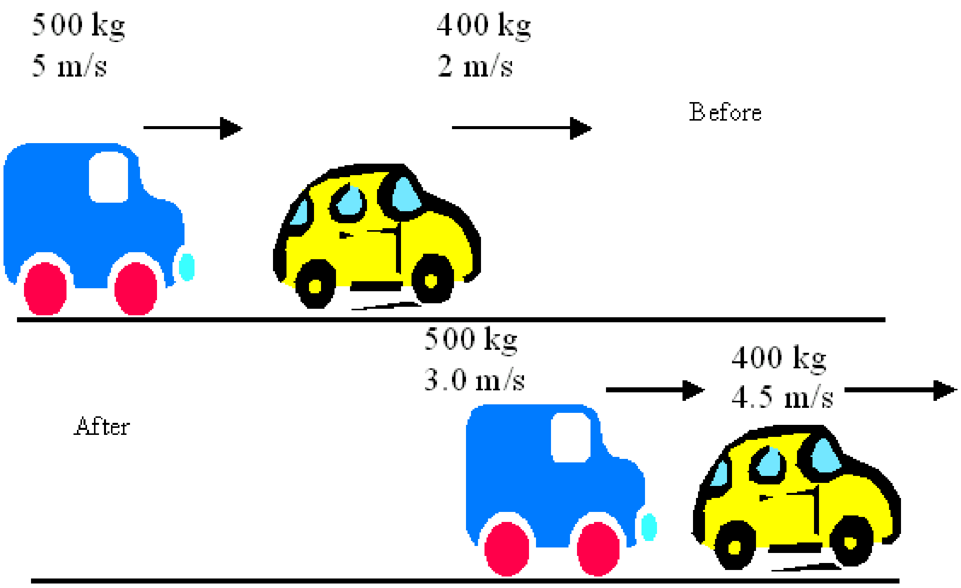
5.12.1

The total momentum of a system remains constant provided that no external forces act on the system.

What do you understand by the statement above?

5.12.2

The diagram shows two cars at a fairground, before and after bumping into each other. One car and driver has a total mass of 500 kg, while the other car and driver has a total mass of 400 kg.



(a) What is

(i) the total kinetic energy before the collision.

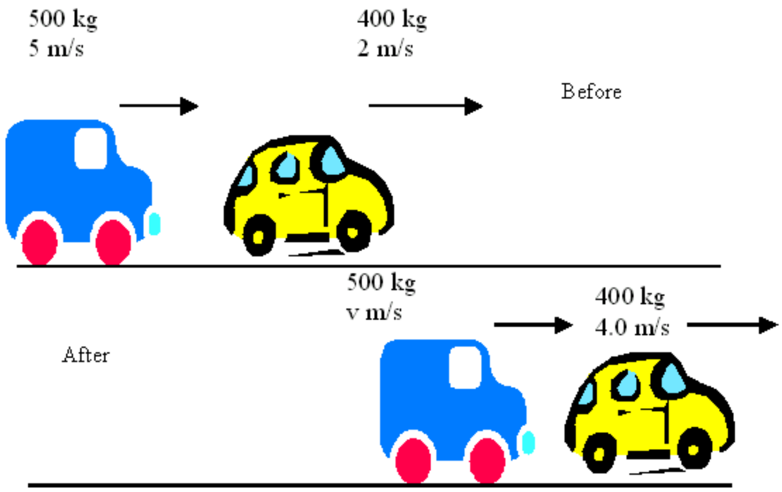
(ii) the total kinetic energy after the collision.

(iii) the total loss in kinetic energy.

(b) Is this an elastic collision? Explain your answer.

5.12.3

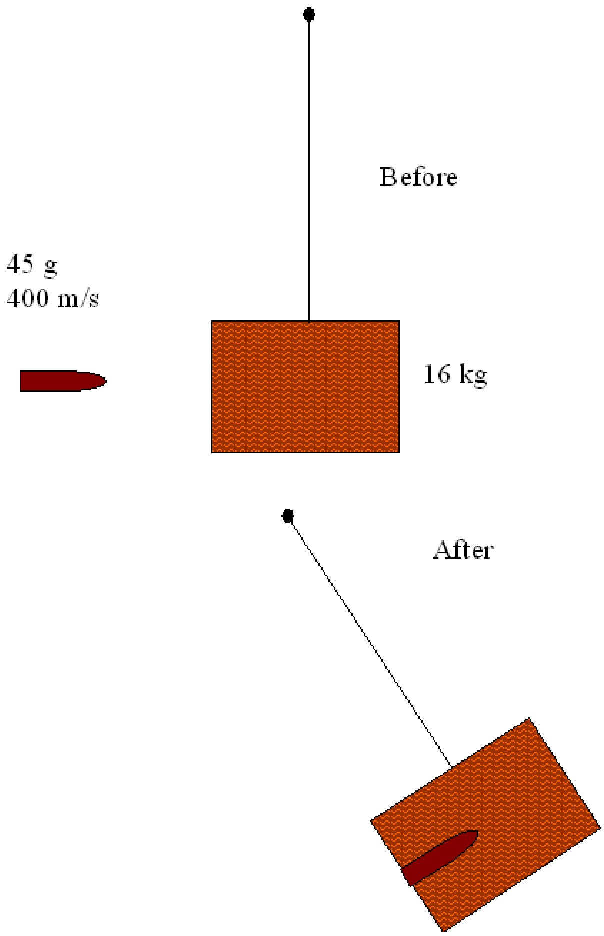
A second collision is shown below:



What is the speed of the 500 kg car after the collision?

5.12.4

A bullet of mass 45 g is travelling horizontally at 400 m/s when it strikes a wooden block of mass 16 kg suspended on a string so that it can swing freely. The bullet is embedded in the block.



Calculate:

a) The velocity at which the block begins to swing.

b) The height to which the block rises above its initial position.

c) How much of the bullet's kinetic energy is converted to internal energy.

5.12.5

A cannon has a mass of 1500 kg. It fires a shell of mass 25 kg. The gunpowder charge is 10 kg. The cannon recoils at a velocity of 10 m/s. What is the velocity of the shell? (The answer is NOT 600 m s^{-1})

5.12.6

A large railway wagon of mass 35 000 kg is travelling from left to right at a velocity of 5.0 m s^{-1}. A small wagon of mass 25 000 kg is travelling at a velocity of 2.0 m s^{-1} from right to left. They collide. The larger wagon then moves off at a velocity of 1.5 m s^{-1} from left to right. Calculate:

(a) The velocity of the smaller wagon.

(b) The energy transferred in the collision.

5.12.7

Read the paragraphs below *Figure 148* (pp 179-180)

Where does the kinetic energy for the nuclei come from?

5.12.8

(Challenge for A level only)

Consider the decay of polonium-214 to lead-210:

$$^{214}_{84}\mathrm{Po} \rightarrow {}^{210}_{82}\mathrm{Pb} + {}^{4}_{2}\alpha$$

The binding energy of the polonium is 1666.01 MeV

The binding energy of the lead is 1645.55 MeV

The binding energy of the alpha particle is 28.296 MeV

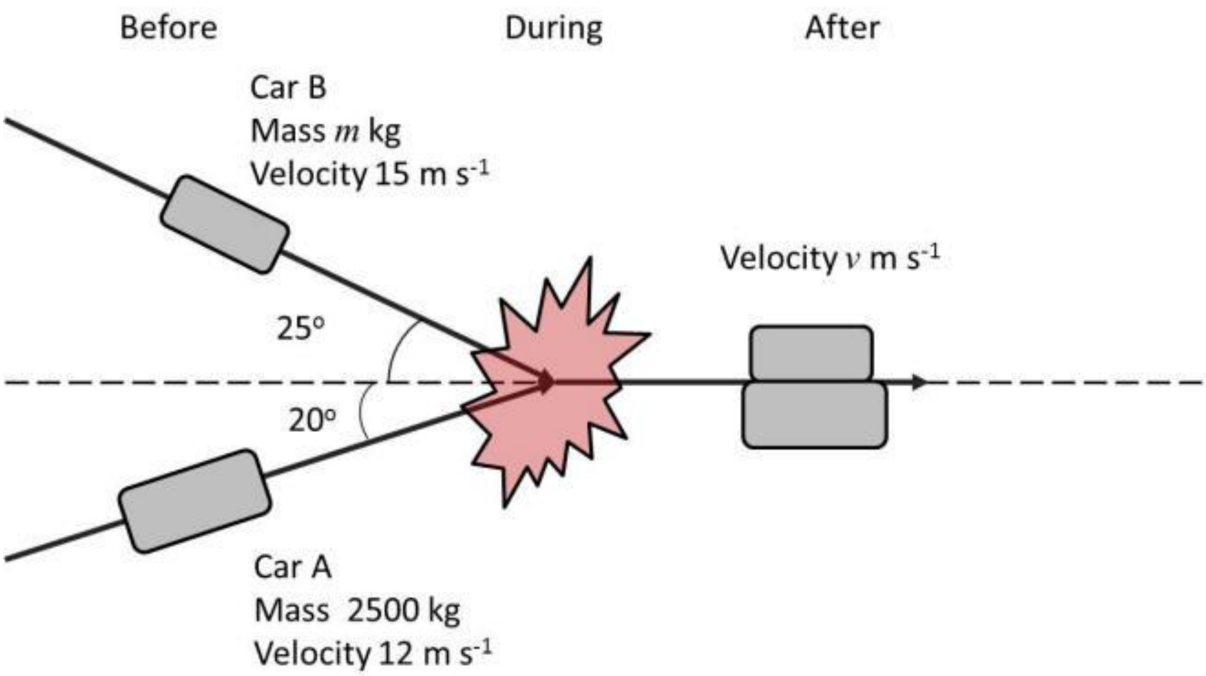
(a) Calculate the Q-value.

(b) Calculate the kinetic energy of the alpha particle in MeV and J.

(c) Calculate the speed of the alpha particle.

5.12.9 (For A-level students of the OCR syllabus only)

Black ice is very dangerous for motorists. The mixture of water and water ice makes for a very slippery surface. Cars A and B are travelling too fast for the conditions and, because of they are travelling on black ice, the drivers cannot control their vehicles. The two cars are approaching a fork where two roads merge into one, as shown on the diagram:



The two vehicles collide and travel in a horizontal line (due East) as shown in the diagram.

Car A has a mass of 2500 kg. Before the collision, it is travelling at 12 m s^{-1} at an angle to the horizontal line of 20°.

Car B has a mass of m kg. Before the collision, it is travelling at 15 m s^{-1} at an angle to the horizontal line of 25°.

After the collision both cars stick together, as shown.

(a) Show that the mass of Car B is about 1600 kg.

(b) Calculate the magnitude of the velocity of the two cars after the collision.

5.12.10

A rocket of mass 1200 kg accelerates vertically upwards off the launch pad with a constant thrust from its engines of 16 kN.

a. Calculate the rocket's weight.

b. Show that the initial acceleration is about 3.5 m s^{-2}.

c. Calculate the rate of loss of mass of the fuel if the exhaust gases are ejected at a speed of 1200 m s^{-1}.

d. Calculate how long the fuel would last if the initial mass of the rocket fuel was 800 kg

e. Calculate the acceleration of the rocket just as the fuel was about to run out.

(Use g = 9.8 m s^{-2})

Part 3 Energy and Work

Tutorial 5.13 Work, Energy, and Power

All Syllabi

Contents

5.131 Work

Work is defined as:

The product of force and the distance moved *(1 mark)* **in the direction of the force** *(1 mark)*.

Work = Force × distance moved in the direction of the force.

The applied force is in the same direction as the movement (*Figure 161*).

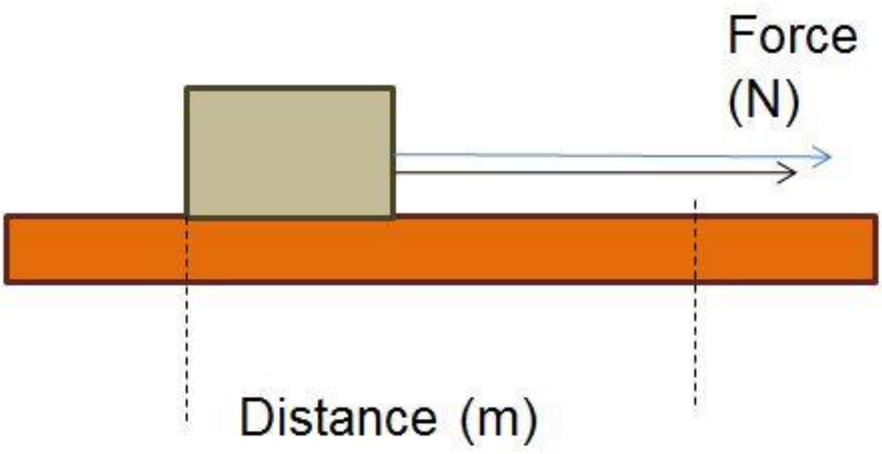


Figure 161Work done in the direction of the force

This uses the formula familiar from GCSE:

$$W = Fs$$

.... *Equation 138*

If the applied force is at an angle the movement, we see (*Figure 162*).

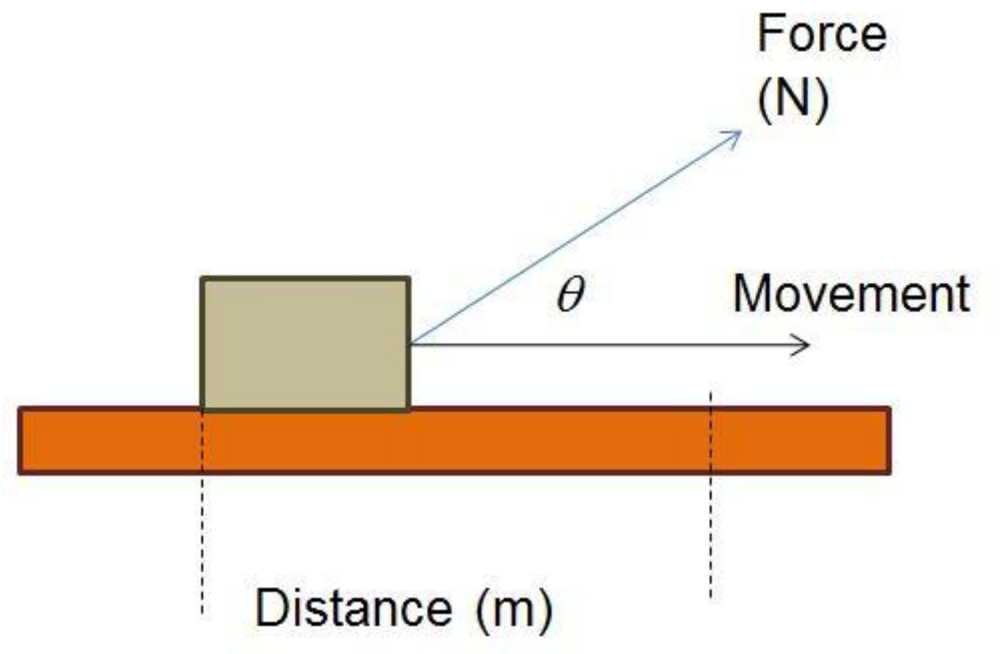


Figure 162 Work done by a force at an angle

This time the formula becomes:

$$W = Fs\cos\theta$$

.... *Equation 139*

Note that:

- Units are **newton metres** (Nm) or **joules** (J).
- Work is actually a **scalar** quantity despite being the product of a vector quantity.
- Normally we consider the line of action of the force and the line of displacement to be at zero degrees to each other. The cosine of zero is 1. However, if we have the line of action of the force and the displacement at an **angle**, we have to use the **cosine function** to take this into account.
- When work is done, there must be movement. This can result in acceleration, a rise in temperature, or deformation in shape.

It is wrong to say that Work = Force × displacement. If we push the box 5 metres and back to where it started, the displacement is 0, but the distance in the direction of the force is 10 metres.

We can plot a graph of force against distance moved (*Figure 163*):

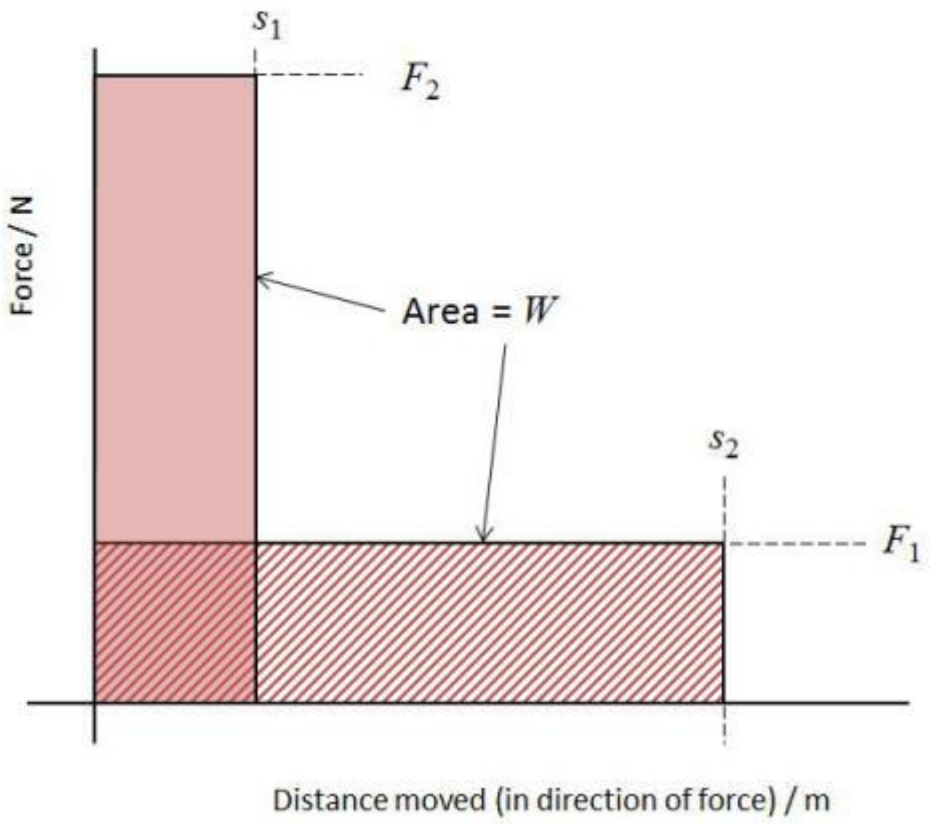


Figure 163 Large force over a small distance results in the same work as a small force over a larger distance.

In this graph we see that a small force (F_1) is applied over a longer distance (s_2) and a larger force (F_2) is applied over a shorter distance (s_1). The areas under both graphs, the work done, is the same, W.

If the force varies as in the graph below (*Figure 164*), the work done is still the area.

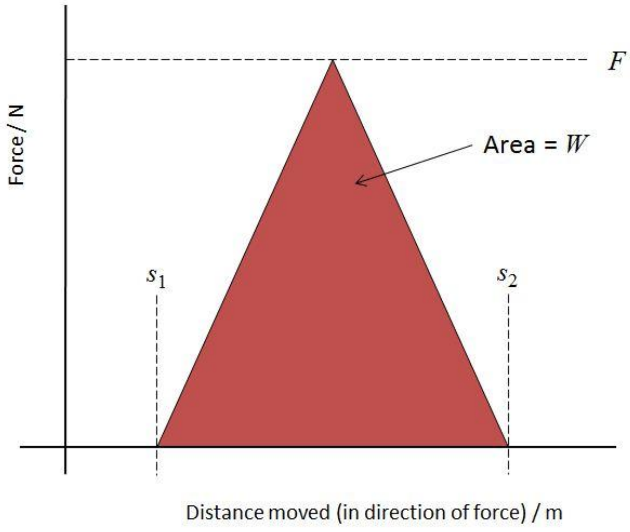


Figure 164 If the force varies, the work done is still the area under the graph.

Be careful here. The start point is not zero, but s_1. The work done = area of the triangle. Therefore:

$$W = \frac{1}{2}F(s_2 - s_1)$$

.... *Equation 140*

If we have an irregular variation of the force, the work done is still the area under the graph (*Figure 165*).

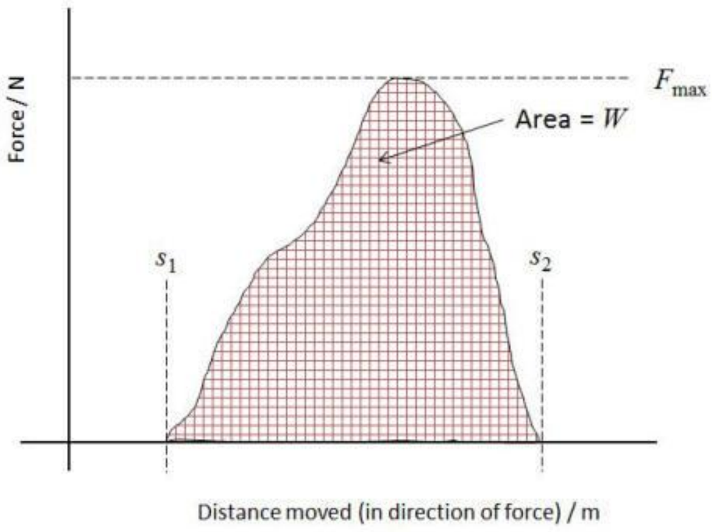


Figure 165 Work done is still the area under the force-distance graph.

In this case we have to count the squares.

5.132 Energy

Energy and work are very closely related.

- Energy is the **ability to do work**. When work is done, energy is transferred.
- Energy comes in many forms.
- Some kinds of energy can be stored, while others cannot.
- Energy is always **conserved**.

5.133 Power

Power is the **rate** at which energy is used or the rate at which work is done.

$$\text{Power} = \frac{\text{energy transferred (J)}}{\text{time taken (s)}} = \frac{\text{work done (J)}}{\text{time taken (s)}}$$

$$P = \frac{\Delta W}{\Delta t}$$

.... *Equation 141*

Units of power are **watt** (W).

- 1 watt = 1 joule per second.
- Also, **kilowatt** (kW). 1kW = 1000 W.
- **Megawatt** (MW). 1 MW = 1×10^6 W.

5.134 Power and Speed

We can also relate power, force and speed:

Work done = force x distance moved.

$$W = Fs$$ .... *Equation 142*

Power = energy ÷ time.

$$P = \frac{\Delta W}{\Delta t}$$

.... *Equation 143*

Speed = distance ÷ time:

$$\frac{\Delta s}{\Delta t} = v$$

.... *Equation 144*

So, we can write *Equation 143*.

$$P = W/t$$

And combine it with *Equation 142* to give *Equation 145*.

$$P = \frac{F\Delta s}{\Delta t}$$

.... *Equation 145*

Therefore, we can write *Equation 146*.

$$P = Fv$$ .... *Equation 146*

Power (W) = **force** (N) × **speed** (m s^{-1})

5.135 Conversion of Work to Energy (Welsh Board)

When work is done, energy is transferred. Energy is needed for us to do a job of work. Assuming no losses, we can say:

Work done = Energy transferred

Consider a toy truck of mass m at rest on a frictionless track (*Figure 166*).

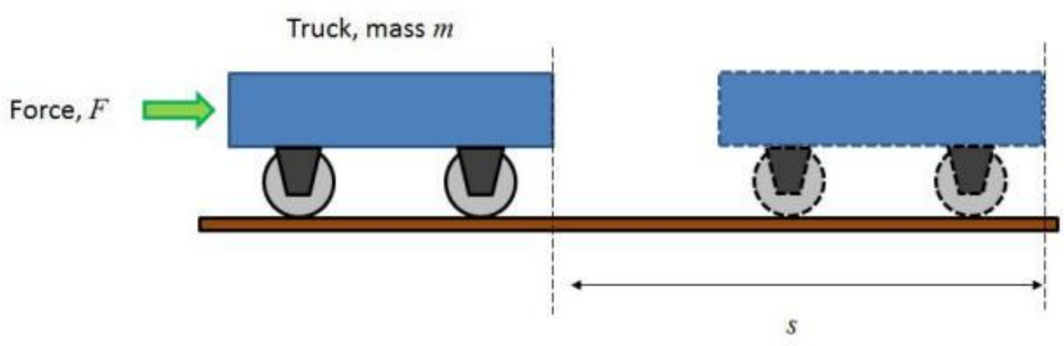


Figure 166 A toy truck being pushed along a frictionless track

A constant force F is applied, and the truck moves through a distance, s. (Note, for consistency, we are using s for distance, rather than x.) We know that:

$$W = Fs \text{ .... } \textit{Equation 147}$$

We know from Newton II that:

$$F = ma \text{ .... } \textit{Equation 148}$$

So, we can easily see that by combining *Equations 147* and *148*, we get:

$$W = mas \text{ .... } \textit{Equation 149}$$

We know also from our equations of motion that:

$$v^2 = u^2 + 2as \text{ .... } \textit{Equation 150}$$

We can ignore the u^2 term in this argument as the truck is at rest, so $u = 0$. We can multiply *Equation 150* by m on both sides:

$$mv^2 = 2mas \text{ .... } \textit{Equation 151}$$

Since $W = mas$ (*Equation 149*), we can say:

$$mv^2 = 2W \text{ .... } \textit{Equation 152}$$

Therefore, rearranging *Equation 152*:

$$W = \frac{1}{2}mv^2 \text{ .... } \textit{Equation 153}$$

Now suppose the truck was travelling at speed u before the force was applied.

We know from our equations of motion that:

$$v^2 = u^2 + 2as$$ .... *Equation 154*

We can multiply the *Equation 154* by m on both sides:

$$mv^2 = mu^2 + 2mas$$ .... *Equation 155*

Since $W = mas$, we can say:

$$mv^2 = mu^2 + 2W$$ .... *Equation 156*

So, we can rearrange *Equation 156* to:

$$W = \frac{1}{2}mv^2 - \frac{1}{2}mu^2$$

.... *Equation 157*

Since $W = Fs$, we can also write:

$$Fs = \frac{1}{2}mv^2 - \frac{1}{2}mu^2$$

.... *Equation 158*

Tutorial 5.13 Questions

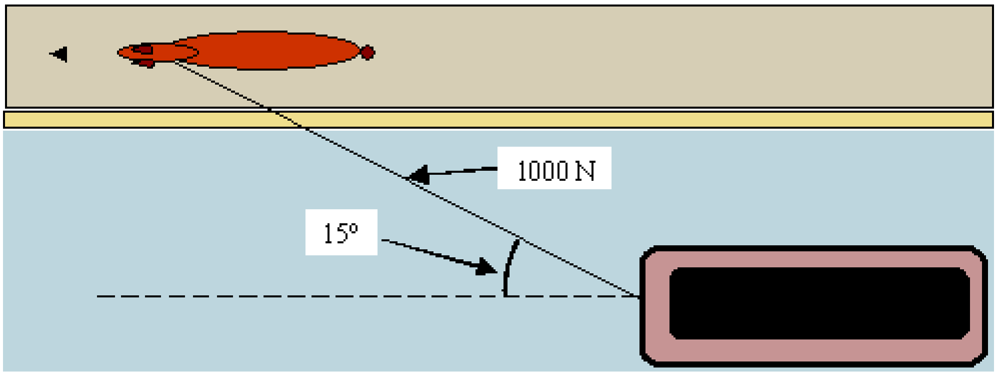
5.13.1

A car owner is trying to bump-start his car, but he cannot get it to move. Sweat is pouring off him. Explain why he has done no work.

5.13.2

A horse is pulling a barge along a canal as shown in the diagram. It pulls the barge with a force of 1000 N a distance of 75 m. The angle the rope is at 15° to the direction of travel.

The situation is shown in the diagram:



(a) Explain why the answer is NOT 75000 J?

(b) What is the work done by the horse?

5.13.3

A box is pushed 5 m across a room with a force of 30 N. What is the work done and how much energy is used?

5.13.4

It takes 20 seconds to push the box in Question 3 across the room. What is the power?

5.13.5

An electric locomotive is pulling a train at a constant speed of 30 m s^{-1}. The train has a rolling resistance of 100 kN.

(a) What force must the locomotive produce? Explain your answer.

(b) What power does it use?

5.13.6

What is the significance of *Equation 153*?

5.13.7

A train of mass 1.50×10^5 kg is travelling at a speed of 20 m s^{-1}.

A constant force of 100 kN is applied to the train over a distance of 500 m.

What is the new speed at which the train is travelling?

Tutorial 5.14 Energy Efficiency	
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5.143 Perpetual Motion	

5.141 Energy Efficiency

No device is ever 100 % efficient. You always have to put more energy in than you get out. This is due to:

- Friction in the moving parts.
- Thermodynamic losses.

All machines lose energy. It is possible to recover a small amount by having, for example, a turbocharger in the exhaust system of a car. Most energy is lost as low-grade heat which is difficult to use. Car engines are much more efficient than they used to be. The efficiency has gone up from about 25 % to 40 %. Even so, for every 100 litres of petrol that you buy, only 40 litres are used to move the car along the road. The rest is used to heat up the environment.

Heat engines depend on a heat gradient from hot to cold, with heat flowing into the environment which is a **heat-sink**.

Energy efficiency is given by the formula with which you will be familiar from GCSE.

$$\% \, Efficiency = \frac{Energy \ Output}{Energy \ Input} \times 100 \ \%$$ .... *Equation 159*

While we normally mention **% efficiency**, it can be expressed as a **fraction**, for example 0.40 instead of 40 %.

5.142 Energy Transfer Diagrams

You will have seen an energy **transfer diagram** at GCSE. The picture below shows a crude version (*Figure 167*).

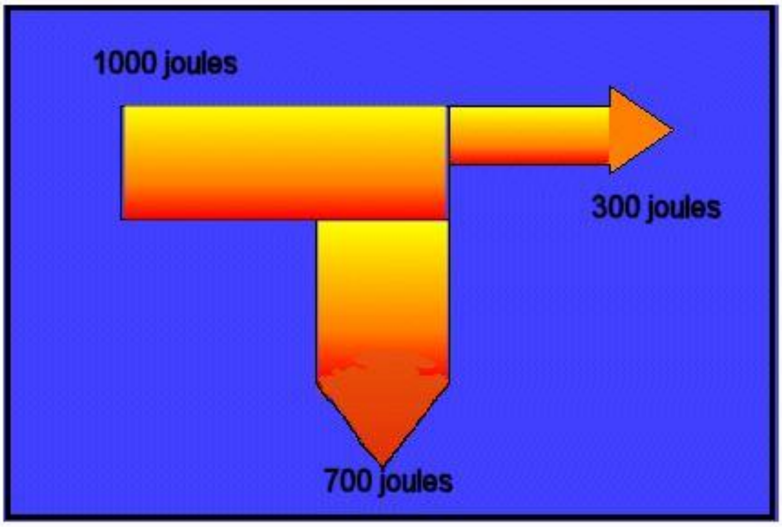


Figure 167 A crude energy efficiency diagram

The arrow that shows the useful energy is the one that goes from left to right. The arrow pointing down shows the energy that is wasted.

Sankey diagrams can also show the quantitative nature of the losses by having a scale. In this diagram we can see that a scale has been used, and the diagram has been plotted on graph paper (*Figure 168*).

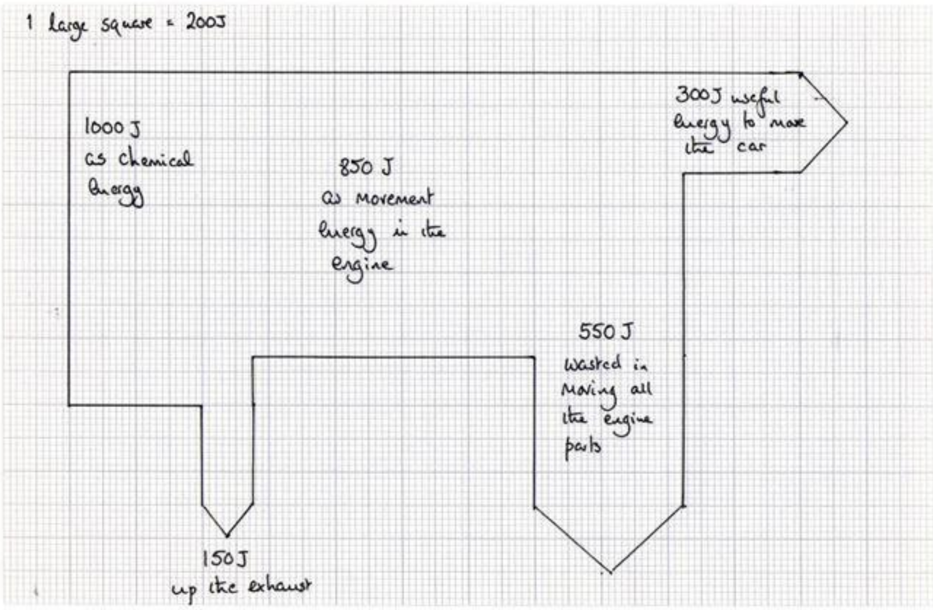


Figure 168 A Sankey diagram

- This diagram shows that for a car, 1000 J is put in (from the chemical energy in the petrol).
- Only 300 J of energy are converted into kinetic energy that moves the car along the road.
- 700 J are wasted as how grade heat to warm up the surroundings. While it is possible to recover heat, for example in the interior heater or a turbocharger, the heat is eventually wasted.

The diagram above is still very simple compared with the kind of diagram shown in *Figure 169*,

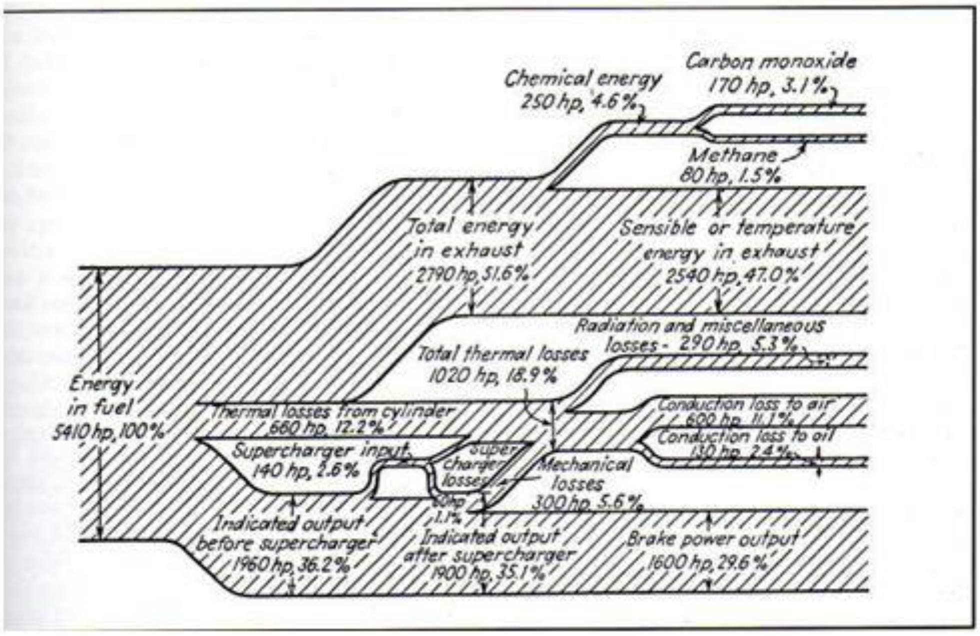


Figure 169 A more complex Sankey diagram

This particular diagram sums up the performance of a large aircraft engine. It gives the power units in **horsepower** (hp) where 1 hp = 746 watts. Nowadays you will see the term **PS** (pferdstärke) used where 1 PS = 750 W.

The basic equation for energy efficiency is shown in *Equation 159*. But here it is again.

$$\text{Efficiency} = \frac{\text{Useful energy (J)}}{\text{Input energy (J)}}$$

This will give a **fraction** that will always be less than 1. We often convert the fraction into a **percentage**, so that:

$$\%\ \text{Efficiency} = \frac{\text{Useful energy (J)}}{\text{Input energy (J)}} \times 100\ \%$$

5.143 Perpetual Motion

If a machine had an efficiency of 100 %, it would be a **perpetual motion machine**. People have been trying to make these for centuries, without success (*Figure 170*).

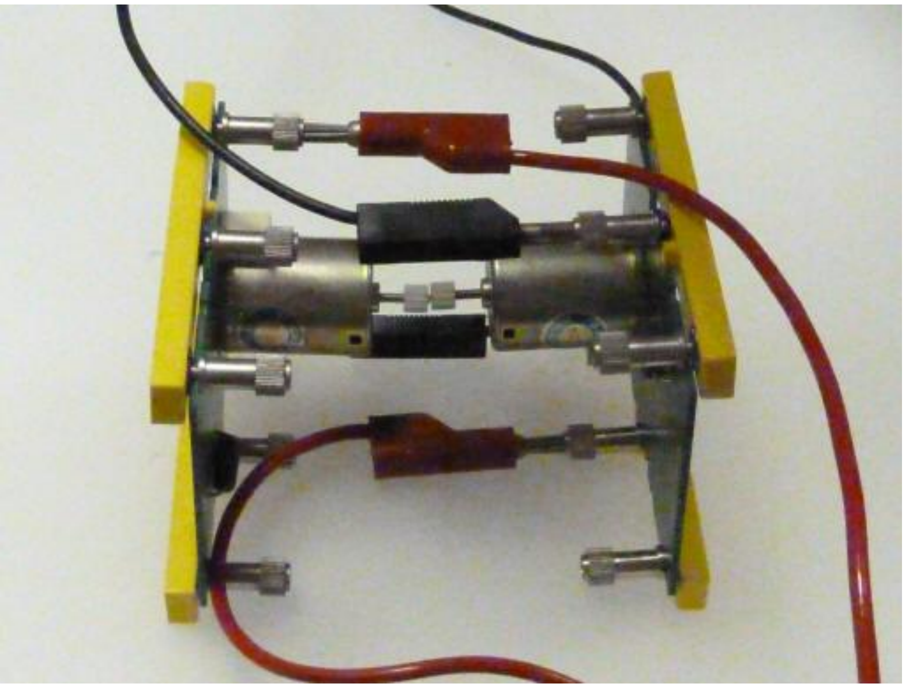


Figure 170 A possible perpetual motion machine.

The picture above shows a possible perpetual motion machine. The motor shafts are joined by glue. The idea of this device is that one of the motors acts as a generator, while the other motor drives the generator. And what the generator produces drives the motor.

It is possible to get "perpetual motion machines" as toys from novelty shops. However they need a battery to keep them going, which is hidden in the base. Once the battery goes flat, they stop.

The study of heat flows is called **thermodynamics**, which is covered in the Optional topic *Engineering Physics*.

Tutorial 5.14 Questions

5.14.1

An electric winch takes 500 A from a 12 V source. It is 25 % efficient. What is its useful power?

5.14.2

Explain why the perpetual motion machine in *Figure 170* will not work.

Tutorial 5.15 Conservation of Energy	
All Syllabi	
Contents	
5.151 Law of Conservation of Energy	5.152 Potential Energy
5.153 Kinetic Energy	5.154 Potential Energy to Kinetic Energy
5.155 Renewable Energy	5.156 Energy in Food

5.151 Law of Conservation of Energy

The Law of Conservation of Energy states:

Energy is neither created nor destroyed; it is converted from one form to another.

At GCSE you would have done some **energy chains**, for example in a nuclear power station (*Figure 171*).

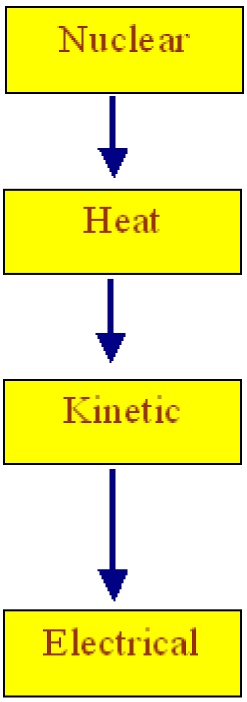


Figure 171 A typical energy chain

Nuclear energy is converted into heat.

Heat boils water to steam.

Heat in the steam is converted into kinetic energy in the turbines.

Which is converted into electrical energy in the generator.

Clanger Corner:
"The law of conservation of energy is a law past by the government to tell us to save energy."

The above nonsense (including spelling mistake) is reproduced from a student's answer in a test! Please be a good chap and don't write drivel like this.

5.152 Potential Energy

This term is often used in the context of **gravitational potential energy**. If we lift an object of mass *m* against gravity, we are doing a job of work.

Work done = PE = weight × distance moved against gravity.

Work done:

$$W = Fs \;\ldots\ldots\ldots \quad \textit{Equation 160}$$

Weight is a force:

$$F = mg \;\ldots\ldots\ldots\ldots \quad \textit{Equation 161}$$

and the distance moved in the direction of the force is the vertical height difference:

$$s = \Delta h \;\ldots\ldots\ldots \quad \textit{Equation 162}$$

The work done is the potential energy, Combining *Equations 160*, *161* and *162* therefore:

$$\Delta E_p = mg\Delta h \;\ldots\ldots\ldots\ldots \quad \textit{Equation 163}$$

Gravitational potential energy is a linear relationship. Therefore, if we plot the gravitational potential energy against the **height difference**, the graph would be like this (*Figure 172*).

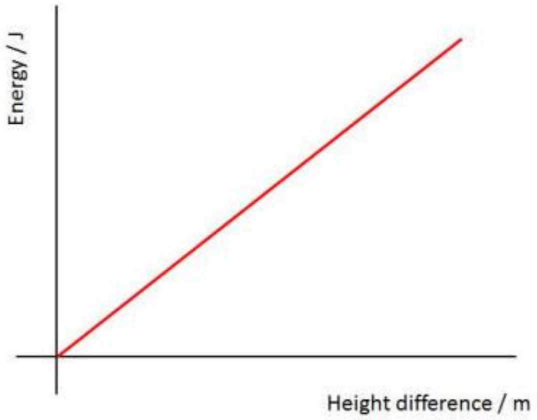


Figure 172 Graph showing proportionality between potential energy and height

In this case the graph shows **direct proportionality**.

This only applies when we are considering the height difference, which we do almost all the time. However, always be careful when you see something like. "...the crane raises a load from a height of 15 m to 25 m". Then you have to work out the height difference - but it's not that hard!

If the energy was plotted against the absolute height, the graph would be linear, but NOT proportional (*Figure 173*).

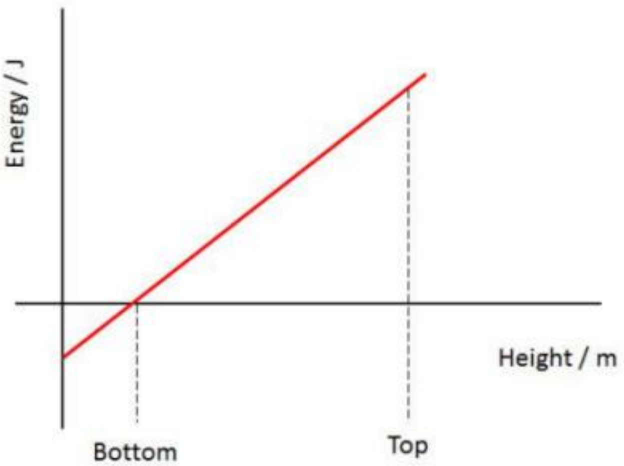


Figure 173 If absolute height is used, the graph is NOT proportional

The potential energy relationship is only true when the gravity field is uniform. This means that the object is **close to the ground**. As we move away from the Earth, we find that the uniform model of the Earth's gravitational field breaks down, and we have to find a different solution. This is discussed when we look at gravity fields from a distance.

Change in gravitational potential is the **change in energy per unit mass**. It is given the physics code ΔV_g, and the units are Joule per kilogram (J kg^{-1}). The change in gravitational potential (ΔV_g) near the Earth's surface is linear, and is given by:

$$\Delta V_g = g\Delta h$$ ………… *Equation 164*

We often use the term **potential energy** in the context of **gravitational potential energy**. There are other kinds of potential energy, for example, **elastic potential energy** in which energy is stored in objects under stress. Objects that are under stress are those that are stretched (under **tension**), squashed (under **compression**) or twisted (under **torque**). We will look at elastic potential energy, or elastic strain energy, in Topic 6, Materials.

Other forms of potential energy include **chemical potential energy** from the bonds of reacting chemicals. Energy is released as heat, that can be made to do work in an engine.

5.153 Kinetic Energy

Kinetic energy is the ability to do work through motion. If the motion is in a straight line, we call the kinetic energy **translational**.

Kinetic energy is given by the equation:

kinetic energy (J) = 1/2 × mass (kg) × (speed (m s^{-1}))^2

In Physics Code:

$$E_k = \frac{1}{2}mv^2$$

…………… *Equation 165*

Suppose we use a force F to accelerate an object steadily from zero up to a speed v. The **kinetic energy** is the **same** as the **work done**.

Kinetic energy (J) = force (N) × distance moved (m) ....*Equation 166*

Since:

Force (N) = mass (kg) × acceleration (m s^{-2}) .... *Equation 167*

we can write:

Force (N) = mass (kg) × [change in speed (m s^{-1}) ÷ time (s)]

In Physics code:

$$F = m \times [(v - 0) \div t] = m \times v/t$$ .... *Equation 168*

Now:

distance travelled = average speed × time

In Physics code:

$$d = [(v - 0) \div 2] \times t = 1/2\ v \times t$$ .... *Equation 169*

Combining *Equations 166* and *169*:

$$E_k = F \times d$$

$$E_k = m \times v/t \times 1/2\ v \times t$$

The t terms cancel out to give us:

$$E_k = 1/2\ mv^2$$ ……………… *Equation 170*

The ability to follow this **derivation** is the sort of thing that will distinguish the outstanding student (A*) from the excellent student (A).

Note that we have talked about **speed** rather than velocity. The kinetic energy of an object is the same **regardless** of direction. We can say this, because of we have a negative velocity, the kinetic energy will still be positive, since minus times a minus is a plus.

5.154 Potential Energy converted to Kinetic Energy

If an object falls, the potential energy is turned into kinetic energy. Then we combine the equations for E_p and E_k (conservation of energy):

$$E_p = E_k \quad \text{............ } \textit{Equation 171}$$

$$mg\Delta h = \tfrac{1}{2}\, mv^2 \quad \text{............ } \textit{Equation 172}$$

$$mg\Delta h = \tfrac{1}{2}\, mv^2 \text{ [masses cancel]} \quad \text{............ } \textit{Equation 173}$$

$$\Rightarrow v^2 = 2g\Delta h \quad \text{........... } \textit{Equation 174}$$

5.155 Renewable Energy

Many chemical energy resources such as **fossil fuels** have the advantage that their energy is in very concentrated form. 50 litres of diesel fuel can take a car 1000 km. They have the two main disadvantages:

- They are non-renewable. Once the resources have been depleted, that's it.
- They cause pollution. Carbon dioxide is responsible for global warming. Sulphur dioxide causes acid rain.

Electric cars do not have these problems. However, their range is limited by their batteries to about 200 – 300 km at best. Electric cars are nothing new; they were around long before petrol cars. However, the limitations of the batteries have always been a problem. Modern batteries give a greater capacity but are very expensive. It is proposed to replace all petrol and diesel vehicles by 2040. However, there is the small problem of infrastructure to allow every house to charge their electric vehicle - it costs money. (If a government chickens out at the cost of erecting 3000 overhead power line masts to electrify the 200 km railway between Kettering and Sheffield, then what hope is there for them to wire fifteen million homes?)

Fossil fuels are **primary** energy sources. A **primary energy source** is one that occurs naturally. Examples include:

- Crude oil.
- Coal.
- Sunlight.
- Nuclear fuel.

Petrol for cars is refined from crude oil and is described as a **secondary energy source**. These renewable energy sources are secondary, with their primary driver being sunlight:

- Wind.
- Solar power.
- Hydroelectric power.
- Wave power.

Tidal power gets its primary energy from the gravitational pulls from the Moon and the Sun.

Most renewable energy projects centre on the generation of **electricity**. Electricity is a **secondary source** that is a particularly convenient to transmit, especially over long distances, and easy to use. It is flexible and can do jobs that other ways of transmission cannot do.

Each has its advantages and disadvantages, as shown in the table below:

Renewable Source	***Advantage***	***Disadvantage***
Hydro-electric	Uses running water of which there is plenty. Renewable. Can be turned on and off quickly	Usually needs a dam to block off a valley to make a reservoir.
Wind	Quiet Relatively inexpensive No emissions. Can be set up in any windy spot.	Useless on a calm day Cannot work if the wind is too strong Often not welcomed by local people. Can disturb birds on migration.
Solar Cells	The intensity of the Sun is 500 W m^{-2} Huge amounts of energy can be harvested.	Solar cells are expensive. Use scarce resources Inefficient. Need large panels Not that effective on dull days.
Solar heating	Sun's rays can be concentrated with mirrors. Lots of heat in a small space.	Needs large mirrors. Mirrors need to track the sun. Does not work well on a cloudy day. Potentially dangerous with intensely concentrated rays of light
Solar panels	Can be placed on any roof of a house. Make lots of hot water.	Water needs to be stored. Ineffective on a cloudy day.

Biomass	New plants can be grown to replace the fuel	Valuable agricultural land taken up for biomass crops. Low energy concentration of biomass fuels.
Biogas	Uses waste material such as rubbish and sewage. Unlimited supply of these materials.	People don't like using it as they are put off by the thought of the source.
Tidal	Large amounts of power from tidal flows Will be there as long as the Moon's there (and nobody is going to take it away).	Environmental considerations, e.g. loss of habitat for wading birds. Power produced according to tides, not when people want it. Very expensive.
Waves	A lot of energy available in waves. Useful for countries with long coastlines.	Machines need to be set up in hostile environments. Often damaged by storms.
Geothermal	Unlimited heat from the centre of the Earth	On most places, deep drilling has to take place to get rocks of adequate temperature. Steam can leak through cracks in the rocks. Best in volcanic areas.

5.156 Energy in Food

To do everything we need to do, we need **energy**. We get this from our **food**. Clearly if we don't eat enough food, we don't have much energy. Prolonged lack of food leads to **malnutrition** with a variety of different health issues.

The converse of this is that people who eat too much get fat. Excessive fatness leads to **obesity**, which can cause a very large number of health problems, including:

- Diabetes.
- Stroke.
- Heart attacks.
- Joint problems.
- Cancer.

Physicists are not exempt from this (*Figure 174*).

Figure 174 Physics tubby, I wonder who this is.

Food energy is most commonly expressed in *calories*. Rather confusingly the food calorie is 1000 calories. In referring to calories, we will use **kilocalories** (kcal).

1 kilocalorie is the amount of energy required to raise the temperature of 1 kilogram of water by 1 Kelvin.

$$1 \text{ kcal} = 4.2 \text{ kJ} = 4200 \text{ J}$$

Food energy is converted into two different kinds of energy:

- Heat energy to keep us warm and ready for action.
- Kinetic energy, produced when our muscles move.

If we do exercise, we get hot as well us pumping iron.

An active young man requires about 2500 kcal of energy. The figure for a young woman is about 2000 kcal.

The answer to Question 5.15.7 will explain why, if you are trying to lose weight, doing a heavy session in the gym and then having a chocolate bar afterwards is counterproductive.

Not all energy is used by the body. Some food is not digestible, and forms the material produced when we sit on the lavatory.

Tutorial 5.15 Questions

5.15.1

Explain what each of the terms in the equation $\Delta E_p = mg \times \Delta h$ mean.

5.15.2

What is the potential energy of a 12 kg mass raised to a height of 25 m?

Use g = 9.8 m s^{-2}. Give your answer to an appropriate number of significant figures.

5.15.3

Calculate the kinetic energy of a shot-put of mass 4.0 kg thrown by an athlete at a speed of 15 m s^{-1}.

5.15.4

A coin is dropped from the viewing platform of an observation tower 80 m high. How fast will it travel just before it hits the ground?

Explain why you do not need the mass of the coin in the question above.

5.15.5

An active young man requires about 2500 kcal of energy. The figure for a young woman is about 2000 kcal.

Convert these figures to kilojoules.

5.15.6

A tutor has a mass of 80 kg and climbs the stairs from the first floor to the sixth floor of a college building, which has a total height of 21 m. Calculate the energy used in raising the tutor through that height. Convert your answer to kilocalories. Use g = 9.81 N kg^{-1}.

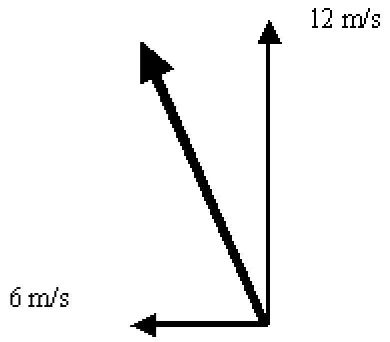
5.15.7

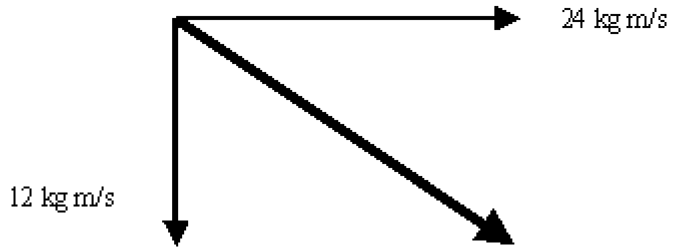
The tutor likes flapjack, which has energy of 400 kcal for 100 g. How much flapjack will provide the tutor with the energy he needs as he goes up the stairs?

Answers to Questions

Tutorial 5.01

5.01.1





Left hand diagram

Resultant velocity: $v^2 = (6 \text{ m s}^{-1})^2 + (12 \text{ m s}^{-1})^2 = 36 \text{ m}^2 \text{ s}^{-2} + 144 \text{ m}^2 \text{ s}^{-2} = 180 \text{ m}^2 \text{ s}^{-2}$

v = **13.4 m s^{-1}**

Right hand diagram:

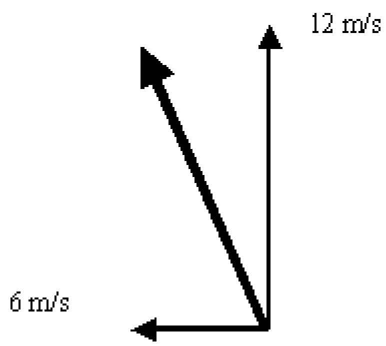
Resultant momentum: $p^2 = (12 \text{ kg m s}^{-1})^2 + (24 \text{ kg m s}^{-1})^2$

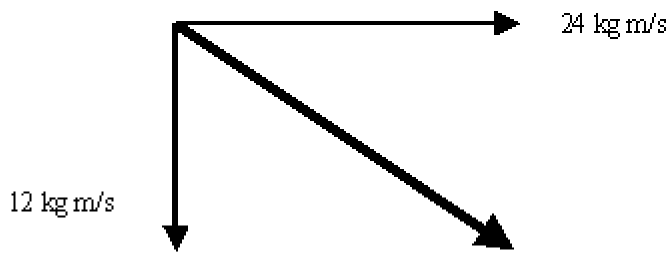
$= 144 \text{ kg}^2 \text{ m}^2 \text{ s}^{-2} + 576 \text{ kg}^2 \text{ m}^2 \text{ s}^{-2} = 720 \text{ kg}^2 \text{ m}^2 \text{ s}^{-2}$

p = 26.8 **kg m s^{-1}**

Make sure that you remember to square root to get your answer. This is a common bear trap.

5.01.2





Left hand diagram

tan θ = opposite ÷ adjacent

$= 6 \text{ m s}^{-1} \div 12 \text{ m s}^{-1} = 0.5$

$\Rightarrow \theta = \tan^{-1} 0.5 =$ **26.6°**

Right hand diagram:

$\tan\theta = 24 \text{ kg m s}^{-1} \div 12 \text{ kg m s}^{-1} = 2.0$

$\Rightarrow \theta = \tan^{-1} 2.0 =$ **63.4°**

Note that when you use the tan function, you can get values of greater than 1. In fact, tan 90° is infinity.

5.01.3

(a) Weight = 1100 kg × 9.8 N kg^{-1} = **10780 N**

(b) Force = 10780 N × cos 10 = 10780 N × 0.985 = **10620 N**

(c) Force = 10780 N × sin 10 = 10780 N × 0.174 = **1870 N**

Tutorial 5.02

5.02.1

(a) $T \cos 10 = 14000 \text{ N} \Rightarrow T = 14000 \text{ N} \div 0.984 = \mathbf{\underline{14200 \text{ N}}}$

(b) $F = 14200 \text{ N} \times \sin 10 = 14200 \text{ N} \times 0.174 = \mathbf{\underline{2470 \text{ N}}}$ (vertically up the picture).

(c) Zero. The ship is not moving; therefore, the force on the wind is balanced by force from the rope.

(d) A vertical force of 2470 N is acting which sums with the 28000 N force.

$$R^2 = (2470 \text{ N})^2 + (28000 \text{ N})^2 = 7.90 \times 10^8 \text{ N}^2$$

$$R = \sqrt{(7.90 \times 10^8 \text{ N}^2)} = 28100 \text{ N} = \mathbf{\underline{28\,000 \text{ N}}} \text{ (2 s.f.)}$$

$$\tan \theta = 2470 \div 28000 = 0.0882 \Rightarrow \theta = \tan^{-1} 0.0882 = \mathbf{5.0}^\circ$$

Did you write 28000 N? The horizontal vector is the resultant of two 14000 N forces.

5.02.2

a. What is the weight of the climber?

W = 68 kg × 9.8 N kg^{-1} = **666 N**

b. Show that the angle θ is about 31°.

12.5 m tan 20 + -7.5 m tan θ = 0

12.5 m tan 20 = 7.5 m tan θ

tan θ = 12.5 m tan 20 ÷ 7.5 m = 0.607

θ = tan^{-1} 0.607 = **31.2°**

c. Show that force T_2 is $1.1 \times T_1$.

$$T_1 \cos 20 = T_2 \cos 31$$

$$T_2 = T_1 \cos 20 \div \cos 31 = 1.096\ T_1 \quad = \mathbf{1.1\ T_1}$$

d. Calculate the values of forces T_1 and T_2.

$$T_1 \sin 20 + T_2 \sin 31 = 666\ \text{N}$$

$$T_1 \sin 20 + 1.1\ T_1 \sin 31 = 666\ \text{N}$$

$$T_1 \times 0.342 + T_1 \times 0.570 = 666\ \text{N}$$

$$T_1 = 666\ \text{N} \div 0.912 = \mathbf{730\ N}$$

$$T_2 = 1.1 \times 730\ \text{N} = \mathbf{803\ N}$$

Tutorial 5.03

5.03.1

The spanner in the picture is 30 cm long and the nut in question has to be tightened to a torque (moment) of 85 N m. What force must the fitter apply?

Moment = Force × distance

85 N m = F × 0.30 m

F = 85 N m ÷ 0.30 m = **283 N**

Confess! Did you forget to convert the distance to metres? A very common bear trap.

5.03.2

The force P is the weight.

Weight = mass × acceleration due to gravity

Weight = 12 kg × 9.8 m s^{-2} = 117.6 N

Weight is a force measured in Newtons. Watch out for this very common bear trap.

Distance OA = 50 cm = 0.5 m

Clockwise moment = 0.5 m × 117.6 N = 58.8 N m

Anticlockwise moment = 58.8 N m = Q × 1.0 m × sin 25 = 0.423 × Q

Q = 58.8 N m ÷ 0.423 = **139 N**

5.03.3

Moment of shelf = 50 N × 0.34 m = 17 N m

Moment from string = 40 N × 0.68 m × $\sin\,\theta$ = 17 N m

$\sin\,\theta$ = 17 N m ÷ (40 N × 0.68 m) = 0.625

θ = 38.7° = **39°** (2 s.f.)

5.03.4

(a) What are the weights of the children?

Use g = 9.8 m s^{-2}:

- A: 35 kg = **343 N**
- B: 25 kg = **245 N**
- C: 40 kg = **392 N**

(b) Child B is sitting 0.4 m from the pivot. Where should child C sit so that the see-saw remains level?

- Anticlockwise moment caused by child A = 343 N × 1.5 m = 514.5 N m
- Total clockwise moment = 514.5 Nm
- Clockwise moment caused by child B = 245 N × 0.4 m = 98 N m
- Clockwise moment caused by child C = 514.5 N m - 98 N m = 416.5 N m
- 416.5 N m = d × 392 N
- d = 416.5 N m ÷ 392 N = **1.06 m**

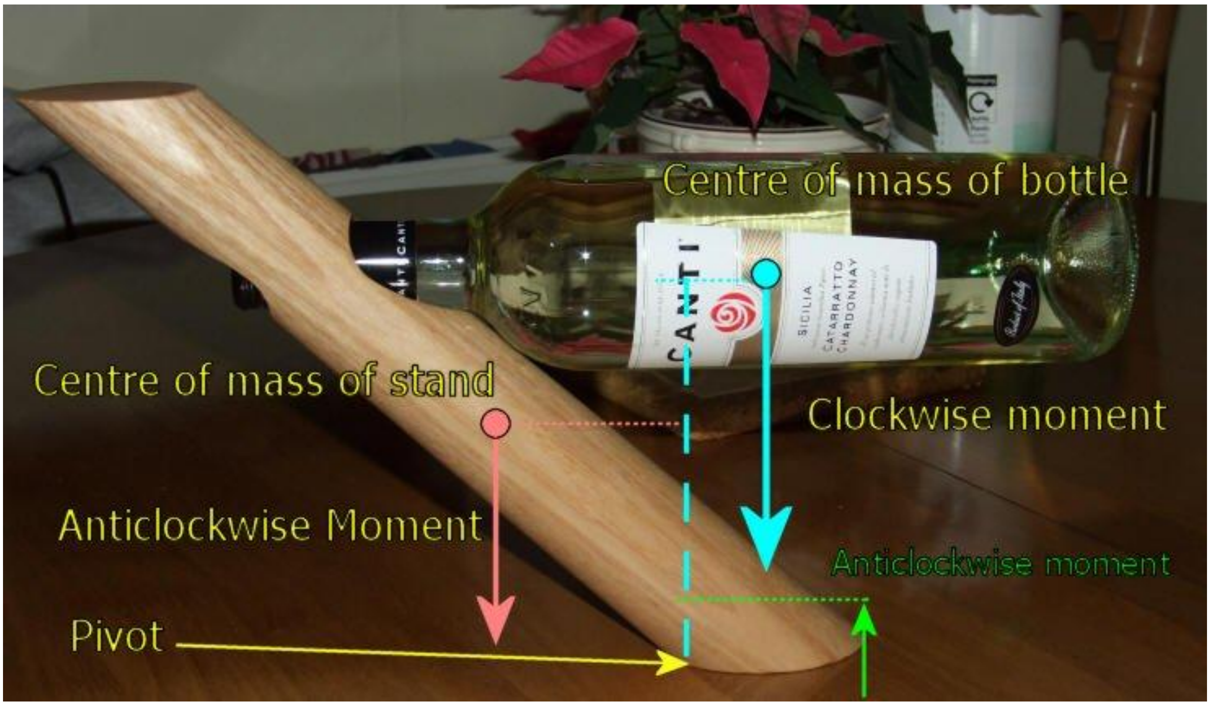
C should sit 1.06 m from the pivot.

(c) Child C misses its footing and falls off the end. What will happen to the others?

- Child A would end up on the floor, because its anticlockwise moment of 515 N m is much larger than the clockwise moment from child B (98 N m)
- Child B would go up in the air.

5.03.5

(a) - (c) Look at the picture:



(d) Use the principle of moments to explain how this system balances.

- The centre of mass of the stand makes an anticlockwise moment.
- The centre of mass of the bottle makes a clockwise moment.
- The clockwise moment of the bottle is bigger than the anticlockwise moment of the stand.
- For the system to be in equilibrium, the anticlockwise moment = clockwise moment.
- Therefore, there has to be another anticlockwise moment…
- …which is provided by the upwards force from the table.

5.03.6

Weight = 50.0 kg × 9.81 N kg^{-1} = 490.5 N

Clockwise moment = 490.5 N × 0.305 m × sin 40 = 96.16 N m

Anticlockwise moment = 96.16 N m = F × 1.10 m

F = 96.16 N m ÷ 1.10 m = **87.4 N** (3 s.f.)

5.03.7

Weight = 20.0 kg × 9.81 N kg^{-1} = 196.2 N

$$S = \frac{mg\cos\theta}{2\sin\theta}$$

S = (196.2 N × cos 72) ÷ (2 × sin 72) = 31.9 N

The length is irrelevant.

Tutorial 5.04

5.04.1

If the height h was the same as d, what is the maximum angle of tilt that the bus could make?

$$\tan\theta = h/d = 1$$

$$\theta = \tan^{-1} 1 = 45^\circ.$$

5.04.2

(a) h = 0.75 m and d = 1.0 m (centre of mass is in the middle)

$\tan \theta = h/d$ = 0.75/1.0 = 0.75

$\theta = \tan^{-1}(0.75)$ = 36.9 °

But this is from the horizontal.

Angle from the vertical = 90 - 36.9 = **53.1°** (QED)

(b)

Moment = $mg \times d \sin \theta$ = 10000 kg × 9.8 N kg^{-1} × 0.75 m × sin θ

= 98000 N × 0.75 m × sin θ

Moment from the wind = 5000 N × 3.0 m = 15 000 N m

⇒ 15000 N m = 98000 N × 0.75 m × sin θ

⇒ sin θ = 15000 N m ÷ (98000 N × 0.75 m) = 0.204

⇒ θ = sin^{-1} (0.204) = **11.5°** (to the horizontal)

(c) The lorry would not tip over, as 11.5° is much less than 37°.

5.04.3

There will be a turning moment to the left as the right wing is much lighter than the left. The pilot would have to steer to the right to raise the left wing.

At best this will make the aeroplane much more difficult to handle. As the pilot slowed down to land, there is a strong chance that the left aileron (a flap which raises the left wing) would become less effective, so the wing dropped right down, causing an uncontrollable spin.

Unfortunately, s/he cannot pull over behind a cloud and transfer the fuel using a petrol can and funnel…

Not the best situation to find oneself in.

Tutorial 5.05

5.05.1

Assume the bridge is uniform.

(a) Weight of the bridge = 20 000 kg × 9.81 N kg^{-1} = **196 200 N**

(b) Moment = 196 200 N × 4.00 m = **784 800 N m**

(c) Force acting on each end of the bridge = 784 800 N ÷ 8.00 m = **98100 N**

5.05.2

The car is 1.50 m from A, while the van is 4.25 m from A.

The car is 4.50 m from B, while the van is 1.75 m from B.

Clockwise moment about A

= (1500 kg × 9.8 N kg^{-1} × 1.50 m) + (10000 kg × 9.8 N kg^{-1} × 3.00 m) + (2000 kg × 9.8 N kg^{-1} × 4.25 m)

= 22050 N m + 294000 N m + 83300 N m = 399350 N m

Anti-clockwise moment about B

= (1500 kg × 9.8 N kg^{-1} × 4.50 m) + (10000 kg × 9.8 N kg^{-1} × 3.00 m) + (2000 kg × 9.8 N kg^{-1} × 1.75 m)

= 66150 N m + 294000 N m + 34300 N m = 394450 N m

Force on A = 394450 N m ÷ 6.00 m = 65.741 kN = **66 kN** (2 s.f.)

Force on B = 399350 N m ÷ 6.00 m = 66.558 kN = **67 kN** (2 s.f.)

(The answers are given to 2 significant figures as *g* is given to 2 s.f.)

Tutorial 5.06

5.06.1

The displacement is zero because we are back at the place we started.

5.06.2

$$\text{Acceleration} = \frac{\text{change in speed}}{\text{time}}$$

Change in speed = speed at end - speed at start = 9.6 m s^{-1} - 0 = 9.6 m s^{-1}

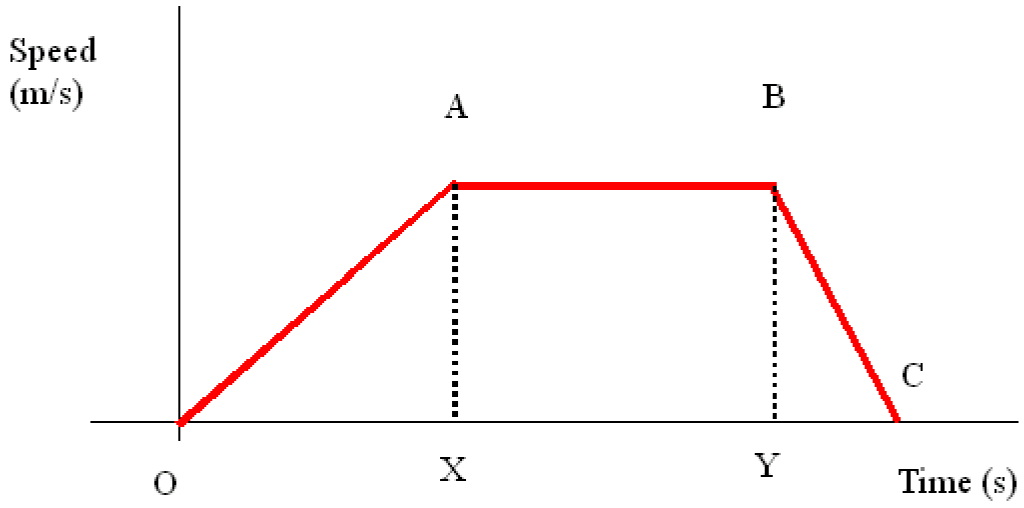
Therefore:

$$4 \text{ m s}^{-2} = \frac{9.6 \text{ m s}^{-1}}{t}$$

Therefore:

$$t = 9.6 \text{ m s}^{-1} \div 4 \text{ m s}^{-2} = \mathbf{2.4\ s}$$

5.06.3



Information:

- The maximum speed of the train is 25 m s^{-1}
- The time interval OX is 45 s
- The time interval XY is 45 s
- The time interval YC is 20 s

What is:

a) The acceleration between O and A:

Gradient of the graph gives the acceleration. Gradient = rise/run

$$\text{Acceleration} = \frac{25\text{ m s}^{-1}}{45\text{ s}} = \mathbf{0.56\text{ m s}^{-2}}$$

b) The acceleration between B and C:

Again, we use the gradient. This time it's negative:

$$\text{Acceleration} = -\frac{25\text{ m s}^{-1}}{20\text{ s}} = \mathbf{-1.25\text{ m s}^{-2}}$$

c) The distance covered while the train is at constant speed:

This time we us the area under the graph. We need to find the area of the rectangle ABXY:

Distance covered = 25 m s^{-1} × 45 s = **1125 m**

d) The total distance.

This time we have to look at the total area under the graph. We already know the area of the rectangle. So, we have to work out the area of the two triangles, OAX and BCY.

Area of OAX = 1/2 × 45 s × 25 m s^{-1} = 562.5 m

Area of BCY = 1/2 × 20 s × 25 m s^{-1} = 250 m

Total distance = 562.5 m + 1125 m + 250 m = 1937.5 m = **1940 m** (3 s.f.)

e) The average speed?

$$\text{Average speed} = \frac{\text{total distance}}{\text{total time}} = \frac{1940\text{ m}}{110\text{ s}} = \mathbf{17.7\text{ m s}^{-1}}$$

5.06.4

A car is travelling at 30 m s^{-1} and takes 10 seconds to accelerate to a new speed of 35 m s^{-1}. What is its acceleration?

Use $v = u + at$

35 m s^{-1} = 30 m s^{-1} + a × 10 s

10 a = 35 m s^{-1} - 30 m s^{-1} = 5 m s^{-1}

a = **0.5 m s^{-2}**

5.06.5

a) The speed of the brick just before it hits the sand.

Use $v^2 = u^2 + 2as$ = 0 + 2 × 9.8 m s^{-2} × 14.5 m = 284.2 m^2 s^{-2}

v = **16.9 m/s**

b) Deceleration in the sand:

Use $v^2 = u^2 + 2as$.

0 = 284.2 m^2 s^{-2} + 2 × a × 0.185 m

Rearranging:

2 × a × 0.185 m = - 284.2 m^2 s^{-2}

$a = \frac{284.2\ m^2\ s^{-2}}{2 \times 0.185\ m}$ = **768 m s^{-2}**

c) What would happen to a person undergoing that deceleration?

768 m s^{-2} is about 77 g, quite sufficient to cause fatal injury.

5.06.6

a

The deceleration = $\frac{0 - 60 \text{ m s}^{-1}}{25 \text{ s}} = - 60 \text{ m s}^{-1} \div 25 \text{ s} = -2.4 \text{ m s}^{-2}$

b

Braking force = mass × acceleration = $5000 \text{ kg} \times -2.4 \text{ m s}^{-2} = 12\,000 \text{ N}$

c

$s = 60 \text{ m s}^{-1} \times 25 \text{ s} + 1/2(-2.4 \text{ m s}^{-2} \times (25 \text{ s})^2) = 750 \text{ m}$.

Did you remember to use the fact that the acceleration was **negative**?

Tutorial 5.07

5.071

a) Use

$$v^2 = u^2 + 2as$$

$v^2 = 0 + 2 \times 1.6 \times 10000 = 32000 \text{ m}^2\text{ s}^{-2}$

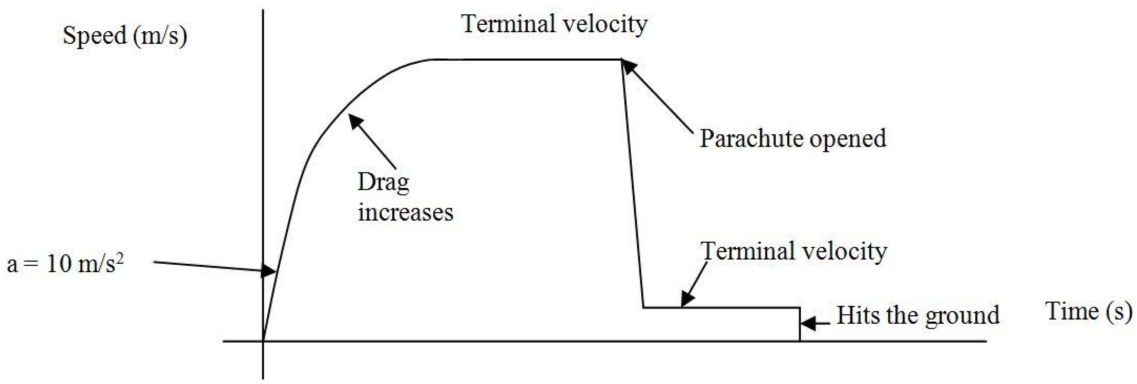
$v = \sqrt{32000} =$ **180 m s^{-1}**

b) Use

$$v = u + at$$

$180 = 0 + (1.6\,t) \Rightarrow t = 180 \div 1.6 =$ **110 s**

5.07.2



5.07.3

When the ball-bearing is released:

- It accelerates under its weight, initially at 9.8 m s^{-2} (for a very short time);
- The viscosity of the liquid is a drag force.
- The drag reduces the downwards force, hence the acceleration reduces in value.
- This increases until the drag force is equal and opposite to the weight.
- There is zero acceleration…
- Hence constant speed, the terminal speed.

5.07.4

(a) The experiment can be carried out like this:

- Use identical cylinders and have exactly the same volume of each oil in each one.
- Have a number of identical ball-bearings.
- Record the mass.
- Measure the radius of each one.
- Drop the ball bearing into the first liquid.
- Time the drop between two fixed points.
- Do repeats to get an average.
- Calculate the terminal velocity.
- Move on to the next oil.

(b) To determine viscosity:

- The greater the time taken, the higher the viscosity.
- If we plot a graph of weight against the product of velocity and diameter.
- Calculate gradient from the graph.
- Gradient is k.
- k is given by this relationship:

$$k = \frac{3\pi\eta}{(1 - \frac{\rho_0}{\rho})}$$

The strange looking symbol η is "eta", a Greek long 'ē'. This is the physics code for viscosity (how gooey a liquid is).

Viscosity of the fluid (N s m^{-2})

Density of steel = 7600 kg m-3

Density of liquid (kg m-3)

If you work out k from the gradient of the graph, and you know the density of the fluid, you can work out the viscosity of the fluid.

You are NOT expected to work with this relationship in the First Year (AS).

Tutorial 5.08

5.08.1

By streamlining, to avoid large flat areas

perpendicular to the direction of travel.

5.08.2

$$F = 1100\text{ kg} \times 9.8\text{ N kg}^{-1} = 10780\text{ N}$$

$$p = \frac{10780\text{ N}}{16\text{ m}^2} = 674\text{ N m}^{-2}$$

The fraction = 674 N m^{-2} ÷ 1.013 × 10^5 N m^{-2} = **6.7×10^{-3} N m^{-2}**

(This is equivalent to 6.7 millibar)

5.08.3

The pilot has deployed the airbrake...

...and lowered the flaps.

5.08.4

Kinetic energy $\propto$ speed^2

Double the landing speed, you have to dissipate 4 times the energy.

This will make the brake discs very hot.

5.08.5

Power = force × speed

Drag force $\propto$ speed^2.

Therefore, power $\propto$ speed^3.

5.08.6

It would have to be 8 times the power, which is 800 PS or 600 kW.

5.08.7

Weight = 2.45 N × 9.81 N kg^{-1} = 24.03 N

$$\mu = \frac{12.6\ \text{N}}{24.03\ \text{N}} = 0.524$$

5.08.8

$$\mu = \frac{6.0\ \text{N}}{6.9\ \text{N}} = 0.870$$

$$\theta = \tan^{-1} 0.870 = 41°$$

Tutorial 5.09

5.09.1

The girl throws the ball at an upward velocity of 15 m s^{-1}. How high will it go?

Use $v^2 = u^2 + 2as$

Upwards is positive, downwards negative.

$0 = 15^2\ m^2\ s^{-2} + 2 \times -9.8\ m\ s^{-2} \times s$

$-19.6\ m\ s^{-2} \times s = -225\ m^2\ s^{-2}$

$s = 225\ m^2\ s^{-2} \div 19.6\ m\ s^{-2} = 11.48\ m =$ **11 m** (2 s.f.)

5.09.2

How long will it take the ball to reach its maximum height?

Use $v = u + at$

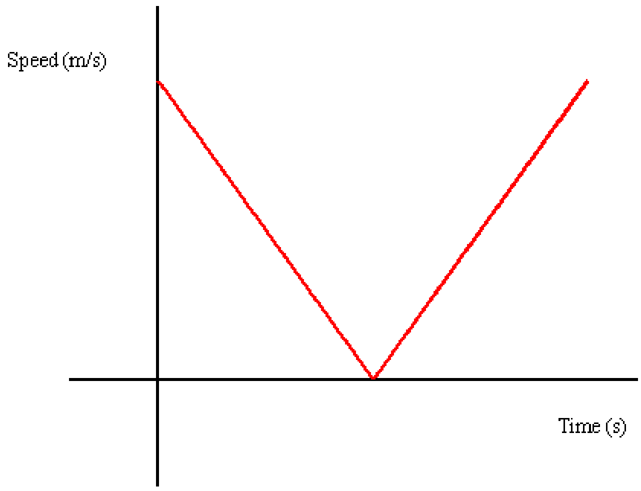
Upwards is positive, downwards negative.

$0 = 15\ m\ s^{-1} + -9.8\ m\ s^{-2} \times t$

$-15\ m\ s^{-1} = -9.8\ m\ s^{-2} \times t$

$t = 15\ m\ s^{-1} \div 9.8\ m\ s^{-2} = 1.53\ s =$ **1.5 s** (2 s.f.)

5.09.3



The graph looks like this because **speed is the just the value**. It does not take into account the direction.

5.09.4

The velocity remains a constant 40 m/s (from left to right).

5.09.5

Use $v^2 = u^2 + 2as$

v^2 = 0 + 2 × 9.8 m s^{-2} × 100 m = 1960 m^2 s^{-2}

v = **44.3 m s^{-1}** (which is about 44 m s^{-1})

Confess!

Did you put 40 m s^{-1} into the u term?

5.09.6

Use Pythagoras

Resultant^2 = (40 m s^{-1})^2 + (44.3 m s^{-1})^2 = 1600 m^2 s^{-2} + 1960 m^2 s^{-2} = 3560 m^2 s^{-2}

Resultant velocity = 59.7 m s^{-1} = **60 m s^{-1}** (2 s.f.)

5.09.7

1. Work out the horizontal velocity.

Horizontal velocity = $v \cos \theta$ = 25 m s^{-1} × cos 40 = 25 m s^{-1} × 0.766 = **19.2 m s^{-1}**

2. Work out the initial vertical velocity:

The initial vertical velocity = $v \sin \theta$ = 25 m s^{-1} × sin 40 = 25 m s^{-1} × 0.643 = **16.1 m s^{-1}**

3. Now work out the time it takes to get to the maximum height:

0 = 16.1 m s^{-1} + -9.8 m s^{-2} × t (=> 0 - 16.1m s^{-1} = -9.8 m s^{-2} t)

t = 16.1 m s^{-1} ÷ 9.8 m s^{-2} = 1.64 s

Therefore, the total time in the air = 2 × 1.64 s = **3.28 s**

4. Therefore, the range = $v \cos \theta \times 2t$

= 3.28 s × 19.2 m s^{-1} = **63 m**.

Tutorial 5.10

5.10.1

(a) Why is the graph a straight line at low speeds?

This is because the opposing forces (friction and drag) are very much less than the driving force from the engine. Therefore, the speed increases at a constant rate from the constant force provided by the engine.

(b) Use Newton I to explain why the car reaches a maximum speed which it cannot exceed. Assume it's on a test track, so it can exceed the 115 km/h national speed limit.

The opposing forces increase so that the balance out the force provided by the engine. Therefore, the car will travel at a constant speed.

5.10.2

A 70 kg athlete accelerates to his maximum speed of 9.5 m/s in a time of 2.5 s. What is the average force he applies to the track?

Work out the acceleration first:

$$v = u + at$$

$$9.5 \text{ m s}^{-1} = 0 + a \times 2.5 \text{ s}$$

$$a = 9.5 \text{ m s}^{-1} \div 2.5 \text{ s} = 3.8 \text{ m s}^{-2}$$

Now use $F = ma$

$$F = 70 \text{ kg} \times 3.8 \text{ m s}^{-2} = \underline{\mathbf{266\ N}}$$

5.10.3

A locomotive of mass 100 tonnes is hauling a train of wagons of mass 1200 tonnes with a pulling force (tractive effort) of 180 kN. What is the acceleration of the train?

First of all, do the conversions:

mass of the locomotive = 100 t × 1000 kg t^{-1} = 100 000 kg

mass of the train = 1200 t × 1000 kg t^{-1} = 1 200 000 kg

pulling force = 180 000 N

Now find the total mass of the train:

Mass = mass of locomotive + mass of wagons

mass = 100 000 kg + 1 200 000 kg = 1 300 000 kg

Now use $F = ma$

180 000 N = 1 300 000 kg × a

$a = \frac{180\,000\text{ N}}{1\,300\,000\text{ kg}}$ = **0.14 m s^{-2}**

5.10.4

(a)

Forwards force, *F* = 1000 N × cos 15 = 1000 N × 0.966 = 966 N

Total mass = 5000 kg + 800 kg = 5800 kg

a = 966 N ÷ 5800 kg = 0.167 m s^{-2} = **0.17 m s^{-2}** (2 s.f.)

(b) Use $v = u + at$

1.5 m s^{-1} = 0 + 0.167 m s^{-2} t

t = 1.5 m s^{-1} ÷ 0.167 m s^{-2} = 8.98 s = **9.0 s**

5.10.5

One problem is that students will not add the mass of the weight to the mass of the trolley.

The second problem is that the heavier the mass loaded onto the string, the relationship with acceleration is no longer linear. It tends towards a maximum of 9.8 m s^{-2}.

Tutorial 5.11

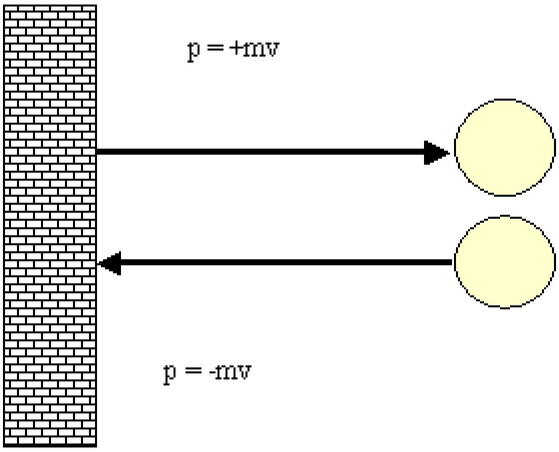
5.11.1

What is the value of the momentum of a 10 kg ball running down a bowling alley at a speed of 5 m/s?

Formula first: $p = mv$

p = 10 kg x 5 m s^{-1} = **50 kg m s^{-1}**

5.11.2



Change in momentum = momentum after - momentum before.

Change in momentum = $+ mv - - mv = + 2mv$

5.11.3

Change in momentum = momentum after - momentum before.

Change in momentum = $+ mv - - mv = + 2mv$

Change in momentum = + 0.2 kg x 6 m s^{-1} - - 0.2 kg x 6 m s^{-1} = **+2.4 kg m s^{-1}**

5.11.4

The formula says that the force = change in momentum / time interval.

Change in momentum = momentum after - momentum before.

In Physics code we write: $\Delta p = mv - mu$

We can write the whole thing in physics code:

$$F = \frac{m(v-u)}{\Delta t}$$

$v-u$ is change in velocity, and change in velocity / time interval = acceleration

So, we can write Force = mass x acceleration which is commonest expression of Newton II

5.11.5

a) Calculate the average force exerted on the driver by his seat belt.

First, we need to work out the change in momentum (impulse):

$\Delta p = mv - mu$ = 0 - 85 kg x -24 m s^{-1} = **2040 N s**

Now we can work out the force:

$F = \Delta p/\Delta t$ = 2040 N s ÷ 0.400 s = **5100 N**

b) Compare this force to his weight and hence work out the "g-force"

The weight of the driver = 85 kg x 9.81 N kg^{-1} = 834 N

The g-force = 5100 N ÷ 834 N = **6.1 g** (i.e. about 60 m s^{-2})

c) Comment on the likelihood of serious injury.

This will reduce the likelihood of serious injury as the body withstand accelerations of up to 8 g. Aerobatic pilots regularly pull 5 to 6 g in their manoeuvres.

5.11.6

The change in momentum is 2040 N s. The mass of the driver is 85 kg.

Change in kinetic energy = work done = (2040 N s)^2 ÷ (2 × 85 kg) = 24480 J

= **24 000 J** (to 2 s.f.)

Tutorial 5.12

5.12.1

The different momenta will have different values and different directions.

When these are summed, they add up to zero.

5.12.2

(a) What is

(i) the total kinetic energy before the collision.

- Kinetic Energy = $1/2\ mv^2$
- Kinetic Energy of blue car = 1/2 x 500 kg x (5.0 m s^{-1})^2 = 6250 J
- Kinetic Energy of blue car = 1/2 x 400 kg x (m s^{-1})^2 = 800 J
- Total energy = 6250 J + 800 J = **7050 J**

(ii) the total kinetic energy after the collision.

- Kinetic Energy = $1/2\ mv^2$
- Kinetic Energy of blue car = 1/2 x 500 kg x (3.0 m s^{-1})^2 = 2250 J
- Kinetic Energy of blue car = 1/2 x 400 kg x (4.5 m s^{-1})^2 = 4050 J
- Total energy = 2250 J + 4050 J = **6300 J**

(iii) the total loss in kinetic energy.

Total loss = 7050 J - 6300 J = **750 J**

(b) Is this an elastic collision? Explain your answer.

It is not elastic (or it's inelastic), because there has been energy lost.

5.12.3

What is the speed of the 500 kg car after the collision?

- Momentum before = momentum after
- Momentum before = $m_1u_1 + m_2u_2$
- Momentum before = (500 kg x 5.0 m s^{-1}) + (400 kg x 2.0 m s^{-1})
- Momentum before = 2500 kg m s^{-1} + 800 kg m s^{-1} **= 3300 kg m s^{-1}**

3300 kg m s^{-1}= 500 kg × v m s^{-1} + 400 kg × 4.0 m s^{-1}

3300 kg m s^{-1} – 1600 kg m s^{-1} = 500 v

v = 1700 kg m s^{1} ÷ 500 kg

v = **3.4 m s^{-1}**

5.12.4

Calculate:

a) The velocity at which the block begins to swing.

- Momentum before = momentum after
- Momentum before = $m_1u_1 + 0$
- Momentum before = (0.045 kg x 400 m s^{-1}) + (0)
- Momentum before = 18 kg m s^{-1}

- Momentum after = total mass of bullet and wood x speed
- Momentum after = (16.000 kg + 0.045 kg) x v m s^{-1}
- Momentum after = 16.045 v kg m s^{-1}

Therefore:

16.045 v kg m s^{-1} = 18 kg m s^{-1}

v = 18 kg m s^{-1} ÷ 16.045 = **1.12 m s^{-1}**

b) The height to which the block rises above its initial position.

We use the conservation of energy principle:

- Kinetic energy = potential energy
- $1/2\ mv^2 = mg\Delta h$
- m terms cancel out.

Rearranging:

$\Delta h = \dfrac{v^2}{2g}$ = 1.122 m² s⁻² ÷ (2 x 9.8 m s⁻²) = **0.057 m**

c) How much of the bullet's kinetic energy is converted to internal energy.

We need to know the kinetic energy of the bullet:

$E_k = 1/2\ mv^2$ = 1/2 x 0.045 kg x (400 m $s^{-1})^2$ = 3600 J

Now we need to know the kinetic energy of the block and bullet:

E_k = 1/2 x 16.045 kg x (1.12 m $s^{-1})^2$ = 10.1 J

Kinetic energy lost = 3600 J - 10.1 J = **3590 J**

This energy is not destroyed but converted into internal energy.

5.12.5

Momentum before = 0

Mass of the cannon afterwards is 1500 kg - 25 kg - 10 kg = 1465 kg

Momentum after = 0 = 1465 kg × -10 m s^{-1} + 25 kg × v

25 kg × v = 14650 kg m s^{-1}

v = **586 m s^{-1}** (from left to right)

5.12.6

(a)

Momentum before = 35 000 kg × 5.0 m s^{-1} + 25 000 kg × -2.0 m s^{-1} = 125 000 N s

Momentum after = 125 000 N s = 35 000 kg × 1.5 m s^{-1} + 25 000 kg × v

25 000 v = 125 000 N s - 52500 N s = 72500 N s

v = 72500 N s ÷ 25 000 kg = **2.9 m s^{-1}** (from left to right)

(b)

Energy before = 1/2 × 35 000 kg × (5.0 m s^{-1})^2 + 1/2 × 25 000 kg × (-2.0 m s^{-1})^2

= 437 500 J + 50 000 J = 487 500 J

Energy after = 1/2 × 35 000 kg × (1.5 m s^{-1})^2 + 1/2 × 25 000 kg × (2.9 m s^{-1})^2

= 39 375 J + 105 125 J = 592 625 J

Energy lost = 592 625 J - 487 500 J = 105 125 J = **110 000 J** (2 s.f.)

This is a lot of energy, most of which will be dissipated as heat. Also a loud bang.

5.12.7

The source of the kinetic energy is from within the nucleus. The alpha particle is thought to derive from the edge of the ball of nucleons, and there is a small chance that the group of 4 nucleons strays is in a region where the strong force is less than the electrostatic force. In accelerating the alpha particle, a job of work is done, and the energy for this comes from a small change in the mass defect of the nucleus.

Nuclear energy is transferred to kinetic energy.

5.12.8

(a)

The total energy is given by:

$$Q = (M_D + M_a) - M_P$$

Therefore:

$$Q = (1645.55 \text{ MeV} + 28.296 \text{ MeV}) - 1666.01 \text{ MeV} = 7.836 \text{ MeV}$$

(b)

Kinetic energy of alpha = (210 ÷ 214) × 7.836 MeV = 7.69 MeV

Kinetic energy in J = 7.69×10^{6} eV × 1.602×10^{-19} J eV^{-1} = 1.23×10^{-12} J

(c)

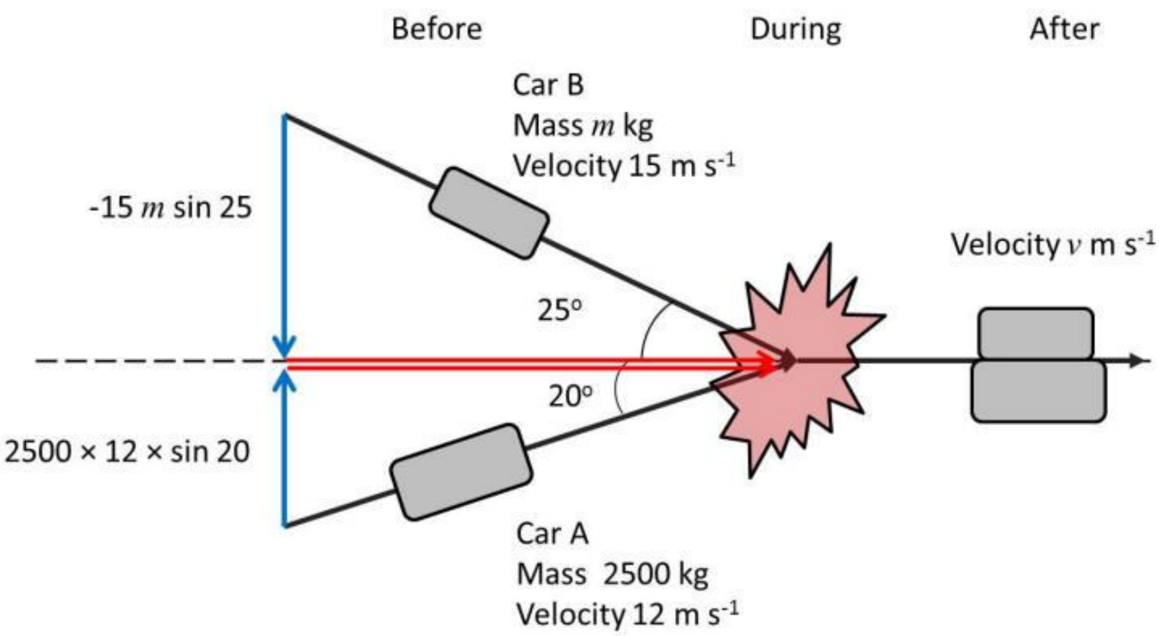
Mass of alpha particle = 1.67×10^{-27} kg × 4 = 6.68×10^{-27} kg.

$v^2 = 2 \times 1.23 \times 10^{-12}$ J ÷ 6.68×10^{-27} kg = 3.68×10^{14} m^2 s^{-2}

v = **1.92×10^{7} m s^{-1}**

5.12.9

(a) Draw a diagram with the vertical and horizontal components of the momenta:



Before the collision Car A has a vertical component

$$= 2500 \text{ kg} \times 12 \text{ m s}^{-1} \times \sin 20 = 10261 \text{ N s.}$$

After the collision, the vertical component for both cars is **zero** as they are travelling on the horizontal line.

As momentum is conserved, the two vertical momenta must add up to zero.

Therefore, Car B has a vertical component of -10261 N s.

Therefore:

$$-15 \text{ m s}^{-1} \times m \times \sin 25 = -10261 \text{ N s}$$

m = 10261 N s ÷ (15 m s^{-1} × sin 25) = **1619 kg** = 1600 kg (QED)

(b) Now work out the horizontal components for the two momenta:

Before the collision Car A has a horizontal component

$$= 2500 \text{ kg} \times 12 \text{ m s}^{-1} \times \cos 20 = 28191 \text{ N s.}$$

Before the collision Car B has a horizontal component

$$= 1619 \text{ kg} \times 15 \text{ m s}^{-1} \times \cos 25 = 22010 \text{ N s.}$$

The two horizontal momenta add up to:

28191 N s + 22010 N s = 50201 N s

The masses of the cars = 2500 kg + 1619 kg = 4119 kg

Now we can work out velocity:

v = 50201 N s ÷ 4119 kg = **12.2 m s^{-1}**

5.12.10

(a)

Weight = 1200 kg × 9.8 N kg^{-1} = **11760 N**

(b)

Resultant force = 16000 N – 11760 N = 4240 N

$a = F/m$ = 4240 N ÷ 1200 kg = **3.53 m s^{-2}** (QED)

(c)

$\Delta p = F\Delta t \Rightarrow$ 1200 kg × m = 16000 N × 1 s $\Rightarrow m$ = **13.3 N s**

(d)

Time taken = 800 kg ÷ 13.3 kg s^{-1} = **60 s**

(e)

Mass of the rocket is 400 kg. Weight = 400 kg × 9.8 N kg^{-1} = 3920 N

Resultant force = 16000 N – 3920 N = 12080 N

a = 12080 N ÷ 400 kg = **30.2 m s^{-2}**

Tutorial 5.13

5.13.1

The car has not moved. Therefore, the displacement is zero.

(Unless you enjoy trouble, don't mention that to the car driver.)

5.13.2

(a) Can you explain why the answer is NOT 75000 J?

The line of action of the force is not in the same direction as the movement. So, we need the horizontal (or more correctly) forward component of the force.

(b) What is the work done by the horse?

$W = Fs \cos \theta$ = 1000 N × 75 m × cos 15 = 1000 N × 75 m × 0.966 = **72 400 J**

5.13.3

A box is pushed 5 m across a room with a force of 30 N. What is the work done and how much energy is used?

Work done = Force × distance moved in the direction of the force = 30 N × 5 m = **150 J**

Energy used = **150 J**

5.13.4

Energy used = **150 J**

Power = energy / time = 150 J ÷ 20 s = **7.5 W**

5.13.5

(a) What force must the locomotive produce? Explain your answer.

It must pull with a force of 100 000 N because the forces are balanced. (We have seen this in Newton's Laws)

(b) What power does it develop?

Power = force × speed = 100 000 N × 30 m s^{-1} = **3 000 000 W** = 3 × 10^6 W = 3 MW

5.13.6

As the force F is applied, the truck will accelerate from rest with constant acceleration. After time t, it will reach speed v.

This shows that the work done is $1/2\ mv^2$. Since there are no losses, we can say that the work done results in transfer to kinetic energy, which we know is:

$$E_k = \frac{1}{2}mv^2$$

5.13.7

Equation:

$$Fs = \frac{1}{2}mv^2 - \frac{1}{2}mu^2$$

Substituting the values:

100×10^3 N $\times$ 500 m = ($1/2 \times 150 \times 10^3$ kg $\times v^2$) - ($1/2 \times 150 \times 10^3$ kg $\times$ (20 m s^{-1})^2)

($1/2 \times 150 \times 10^3$ kg $\times v^2$) = (100×10^3 N $\times$ 500 m) + ($1/2 \times 150 \times 10^3$ kg $\times$ (20 m s^{-1})^2)

($1/2 \times 150 \times 10^3$ kg $\times v^2$) = 5.0×10^7 J + 3.0×10^7 J = 8.0×10^7 J

v^2 = ($2 \times 8.0 \times 10^7$ J) ÷ (150×10^3 kg) = 1067 m^2 s^{-2}

v = **32.7 m s^{-1}** = 33 m s^{-1}

There are other ways of doing this that would be creditworthy.

Tutorial 5.14

5.14.1

Total Power = 500 A × 12 V = 6000 W

Useful power = 0.25 × 6000 W = **1500 W**

5.14.2

In this device both motors will act as generators and will oppose each other.

Even if it were possible to get one of the motors to act as a generator, and the other to drive it, energy will be lost as heat due to:

- resistance in wires.
- friction in the bearings.
- resistance in coils of both motors.

Tutorial 5.15

5.15.1

$\Delta E_p = mg \times \Delta h$

The terms mean:

- Δ - change in.
- E_p - potential energy (J).
- m - mass (kg).
- g - acceleration due to gravity (m s^{-2}).
- Δh - height difference (m).

5.15.2

Potential Energy = weight × height change

Weight = 12 kg × 9.8 N/kg = 117.6 N

Height change = height at end - height at start = 25 - 0 = 25 m

Potential energy = 117.6 N × 25 m = 2940 J = **2900 J** (2 s.f.)

Did you fall into the bear trap of using weight as kilograms?

5.15.3

Kinetic energy = $1/2\ mv^2$

= 1/2 × 4 kg × (15 m $s^{-1})^2$ = 1/2 × 4 kg × 225 m^2 s^{-2} = **450 J**

5.15.4

Potential Energy = Kinetic Energy

$$mg\Delta h = 1/2\ mv^2$$

m terms cancel out

Rearranging:

$$v^2 = 2g\Delta h$$

$$v^2 = 2 \times 10\ \text{m s}^{-1} \times 80\ \text{m} = 1600\ \text{m}^2\ \text{s}^{-2}$$

$$v = \underline{\mathbf{40\ m\ s^{-1}}}$$

We don't need the mass as the masses cancel out.

5.15.5

2000 kcal = 2000 kcal × 4200 J kcal^{-1} = **$\underline{8.4 \times 10^6\ \text{J}}$** = 8400 kJ

2500 kcal = 10.5 MJ = **<u>10500 kJ</u>**

5.15.6

E = 80 kg × 9.81 N kg^{-1} × 21 m = 16500 J

This is 16500 J ÷ 4200 J kcal^{-1} = **<u>3.9 kcal</u>**

5.15.7

Energy used = 3.924 kcal

Energy in flapjack = 4 kcal per gram

Amount needed = 3.924 kcal ÷ 4.00 kcal g^{-1} = **<u>0.981 g</u>**